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THE GROSS-KOHNEN-ZAGIER THEOREM IN HIGHER DIMENSIONS

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1. Introduction. The Gross-Kohnen-Zagier theorem [GKZ] says roughly that the Heegner divisors of a modular elliptic curve are given by coefficients of a vector-valued modular form of weight $3/2$. We give another proof of this (see Theorem 4.5 and Example 5.1), which extends to some more general quotients of hermitian symmetric spaces of dimensions b^- and shows that formal power series, whose coefficients are higher-dimensional generalizations of Heegner divisors, are vector-valued modular forms of weight $1+b^-/2$.

The main idea of the proof of Theorem 4.5 is easy to state. One of the main results of [B] is a correspondence from modular forms of weight $1-b^-/2$ with singularities to automorphic forms with known zeros and poles, which give relations between Heegner divisors and their higher-dimensional generalizations. On the other hand, Serre duality for modular forms says that the only obstructions to finding modular forms of weight $1-b^-/2$ with given singularities are given by modular forms of weight $1+b^-/2$. In other words, the only obstructions to finding relations between Heegner divisors are given by certain modular forms of weight $1+b^-/2$. It is a formal consequence of this that the Heegner divisors themselves are the coefficients of a modular form of weight $1+b^-/2$. The idea of using the results of [B] to prove relations between Heegner divisors was suggested to me by R. L. Taylor.

Most of the more interesting special cases of Theorem 4.5 in low dimensions are already known, though it does at least simplify and unify several previous proofs of known results. For modular curves, the theorem is more or less the same as the main result (Theorem C) of [GKZ], stating that Heegner divisors on modular curves are given by coefficients of a Jacobi form of weight 2. (See Example 5.1.) The main difference is that we prove the result for all Heegner divisors while the authors of [GKZ] restricted their results to the case of Heegner divisors of discriminant coprime to the level for simplicity, though their method could probably be extended to cover all Heegner divisors. The only reason this has not been done before (as far as I know) seems to be that it would take a lot of extra work for a rather small improvement to the result. Hayashi [H] has extended the results of [GKZ] to some of the other discriminants.

There is a similar result for CM points on Shimura curves (Example 5.3). The

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