

SCHUBERT POLYNOMIALS, THE BRUHAT ORDER, AND THE GEOMETRY OF FLAG MANIFOLDS

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To the memory of Marcel Paul Schützenberger

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