Vol. 94, No. 3 DUKE MATHEMATICAL JOURNAL

ON THE LOCATION AND PROFILE OF SPIKE-LAYER SOLUTIONS TO A SINGULARLY PERTURBED SEMILINEAR DIRICHLET PROBLEM: INTERMEDIATE SOLUTIONS

WEI-MING NI, IZUMI TAKAGI, AND JUNCHENG WEI

1. Introduction. In this paper, we continue our investigation [12] on spike-layer solutions to singularly perturbed nonlinear Dirichlet problems. In addition to discussing the existence of "spike-layer" solutions, we study the location of spikes as well as the profile of spikes of the problem

$$\begin{cases} \varepsilon^2 \Delta u + f(u) = 0 & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where

$$\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$$

is the Laplace operator, Ω is a bounded domain in R^n with smooth boundary $\partial\Omega$, $\varepsilon > 0$ is a small parameter, and $f : R \to R$ is of class $C^{1+\sigma}(R)$ with $0 < \sigma < 1$ satisfying the following conditions:

(f1) $f(0) = 0$ and $f'(0) = -m < 0$;
(f2) f has two positive zeros z_1 and z_2 such that $z_1 < z_2, f'(z_2) < 0$, and f has no other positive zeros;
(f3) $\int_0^{z_2} f(s)ds > 0$;
(f4) the function $u \to f(u)/(u - u_0)$ is nonincreasing in the interval (u_0, z_2), where u_0 is defined as the unique number in (z_1, z_2) such that $\int_0^{u_0} f(s)\,ds = 0$.

A typical example for the function f with the properties (f1), (f2), (f3), and (f4), is the "cubic" function $f(u) = u(u - a)(1 - u), 0 < a < 1/2$, which has appeared in various models in applied mathematics, including population genetics and chemical reactor theory. (See, e.g., [7] and the references therein.)

A natural quantity associated with (1.1) is the "energy functional" defined in $W_0^{1,2}(\Omega)$:

Received 9 September 1996.
1991 *Mathematics Subject Classification.* Primary 35B40, 35B45; Secondary 35J40.
Ni's research partially supported by the National Science Foundation.
Wei's research supported by a direct grant from the Chinese University of Hong Kong.