

HERMITE AND SPECIAL HERMITE EXPANSIONS REVISITED

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1. Introduction. On a compact Riemannian manifold M consider a first-order pseudodifferential operator $p(X, D)$ that is positive and selfadjoint. Let λ_j , $j = 0, 1, 2, \dots$, be the sequence of its eigenvalues and $e_j(x)$ the corresponding eigenfunctions. The family $\{e_j\}$ then forms an orthonormal basis for $L^2(M)$. Let $E_j f$ be the projection of f onto the j th eigenspace so that we have

$$f = \sum_{j=0}^{\infty} E_j f, \quad (1.1)$$

where the series converges in the L^2 -norm. For functions f in $L^p(M)$, where p is other than 2, the above series may not converge to f in the L^p -norm, and one is led to consider the Bochner-Riesz means $S_{\lambda}^{\delta} f$.

The Bochner-Riesz means are defined by the equation

$$S_{\lambda}^{\delta} f(x) = \sum_{\lambda_j \leq \lambda} \left(1 - \frac{\lambda_j}{\lambda}\right)^{\delta} E_j f(x), \quad (1.2)$$

and we want to know if $S_{\lambda}^{\delta} f$ converges to f in the L^p -norm as $\lambda \rightarrow \infty$. Let $\delta(p) = \max\{n|1/p - 1/2| - 1/2, 0\}$ be the critical index, where n is the dimension of the manifold. Then a necessary condition for the convergence of $S_{\lambda}^{\delta} f$ to f in the L^p -norm is that $\delta > \delta(p)$. In [6] Sogge proved that this condition is also sufficient as long as $1 \leq p \leq 2(n+1)/(n+3)$ or $p \geq 2(n+1)/(n-1)$. This result includes previously known results for the multiple Fourier series ($M = T^n$, the n -torus) and spherical harmonic expansions ($M = S^n$, the $(n+1)$ -sphere).

Let us leave the premises of compact manifolds and proceed to noncompact situations. The simplest example is the case of the standard Laplacian $-\Delta$ on $\mathbb{R}^n$, and in this case the operator does not have point spectrum. The spectral decomposition is given by the Fourier transform, and one is led to consider the Bochner-Riesz means

$$S_R^{\delta} f(x) = (2\pi)^{-n/2} \int_{|\xi| \leq R} \left(1 - \frac{|\xi|^2}{R^2}\right)^{\delta} \hat{f}(\xi) e^{ix \cdot \xi} d\xi, \quad (1.3)$$

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