COMPUTATION OF THE DIFFERENCE OF TOPOLOGY AT INFINITY FOR YAMABE-TYPE PROBLEMS ON ANNULI-DOMAINS, I

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1. Introduction. Let Ω be a smooth and bounded domain in $\mathbb{R}^n$, $n \geqslant 3$. We consider the nonlinear elliptic problem

$$\text{(I)} \qquad \begin{cases} -\Delta u = u^{(n+2)/(n-2)}, u > 0 & \text{on } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

The interest in this type of equation comes from its resemblance to the Yamabe problem in differential geometry, which consists of finding $u > 0$ satisfying

$$-\Delta u = u^{(n+2)/(n-2)} - \frac{n-2}{4(n-1)} R(x)u \quad \text{on } M,$$

where M is a Riemannian manifold of dimension n without boundary, and $R(x)$ is the scalar curvature (see, for example, [1], [8], [11]).

The special nature of (I) appears when we consider it from the variational viewpoint. Let us define on $H_0^1(\Omega)$ the functional

$$\text{(1)} \qquad J(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 - \frac{n-2}{2n}\int_\Omega |u|^{2n/n-2},$$

whose positive critical points are solutions of (I). The difficulty comes from the possible existence of critical points at infinity, which are orbits of J, along which J remains bounded, the gradient goes to zero, and the orbits do not converge (see [3], [4], [5]).

To prove the existence of critical points of J by studying the difference of topology between the level sets of J, it becomes essential to identify the topological contribution of critical points at infinity, so we need some notation.

We denote by G Green's function of the Laplace operator defined by

$$\text{(2)} \qquad \begin{cases} -\Delta G(x, .) = c_n\delta_x & \text{on } \Omega, \\ G(x, .) = 0 & \text{on } \partial\Omega, \end{cases}$$

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