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ASYMPTOTIC DISTRIBUTION OF EIGENVALUES FOR PAULI OPERATORS WITH NONCONSTANT MAGNETIC FIELDS

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Introduction. In the present work, we study the asymptotic distribution of discrete eigenvalues near the bottom of the essential spectrum for 2- and 3-dimensional Pauli operators perturbed by electric potentials falling off at infinity. Special emphasis is placed on the case in which the Pauli operators have nonconstant magnetic fields.

The Pauli operator describes the motion of a particle with spin in a magnetic field and it acts on the space $L^2(R^3) \otimes C^2$. The unperturbed Pauli operator is given by

$$H_P = (-i\nabla - A)^2 - \sigma \cdot B$$

under a suitable normalization of units, where $A : R^3 \to R^3$ is a magnetic potential, $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ with components

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

is the vector of 2×2 Pauli matrices, and $B = \nabla \times A$ is a magnetic field. We write $(x, z) = (x_1, x_2, z)$ for the coordinates over the 3-dimensional space $R^3 = R_x^2 \times R_z$. Throughout the entire discussion, we suppose that the magnetic field B has a constant direction. For notational brevity, the field is assumed to be directed along the positive z axis, so that B takes the form

$$B(x) = (0, 0, b(x)).$$

Since the magnetic field B is a closed 2-form, it is easily seen that B is independent of the z variable. We identify $B(x)$ with the function $b(x)$. Let $A(x) = (a_1(x), a_2(x), 0)$, with real function $a_j \in C^1(R_x^2)$, be a magnetic potential associated with $b(x)$. Then $b(x) = \partial_1 a_2 - \partial_2 a_1$, $\partial_j = \partial/\partial x_j$, and the Pauli operator takes the simple form

$$H_P = \begin{pmatrix} H_+ - \partial_z^2 & 0 \\ 0 & H_- - \partial_z^2 \end{pmatrix},$$

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