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KUPKA-SMALE THEOREM FOR AUTOMORPHISMS OF $\mathbb{C}^n$

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1. Introduction. The Kupka-Smale theorem is an important and useful tool in the study of dynamical systems. Briefly, this theorem says that for a generic diffeomorphism, all periodic points are hyperbolic, and the stable and unstable manifolds of any two periodic points are transverse. The goal of this paper is to prove this theorem for the space $\mathrm{Aut}(\mathbb{C}^n)$ of biholomorphic automorphisms of $\mathbb{C}^n$ for $n > 1$. The topology on $\mathrm{Aut}(\mathbb{C}^n)$ is the compact-open topology, applied to both a map and its inverse. This topology makes $\mathrm{Aut}(\mathbb{C}^n)$ a Frechet space, and the topology can be induced by a complete metric, which is specified explicitly at the end of Section 3. In particular, Baire's theorem applies to $\mathrm{Aut}(\mathbb{C}^n)$.

To state the Kupka-Smale theorem more precisely, we first recall a few ideas. Writing F^m for m-fold composition of the map F, we say that a periodic point p of minimal period m of a differentiable map F is *hyperbolic* if none of the eigenvalues of $(DF^m)(p)$ has modulus 1. Such a point is called a *saddle point* if $(DF^m)(p)$ has at least one eigenvalue bigger than 1 in modulus and at least one smaller than 1 in modulus. Moreover, p is *linearizable* if there is a neighborhood of p such that F^m is conjugate to a linear map in this neighborhood.

Given a saddle point of period m, there is an associated immersed manifold $W^s(p)$ through p which is invariant under F, which is tangent to the eigenspace $E^s(p)$ associated with the set of eigenvalues with norm less than 1, and which is defined by

$$W^s(p) := \left\{ q : \lim_{k\to\infty} (F^m)^k(q) = p \right\}.$$

This is called the *stable manifold* at p. There is also a local version of this manifold, defined later, and corresponding versions of the unstable manifold when F is invertible or locally invertible. We write $W^s(p,F)$ and $W^u(p,F)$ when we need to specify the map involved.

Finally, a set is *residual* in a space X if it contains a dense $\mathscr{G}_\delta$ subset of X. See [Sh2] for more background on these and related ideas.

In the case of diffeomorphisms, the Kupka-Smale theorem states that there is a residual set of diffeomorphisms such that all of the periodic points are hyperbolic, and such that the stable and unstable manifolds of any two saddle periodic

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