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THE SCALAR-CURVATURE PROBLEM ON HIGHER-DIMENSIONAL SPHERES

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Let (S^n, c) be an n-dimensional sphere with a standard metric. Let K be a C^2-positive function on S^n. The Kazdan-Warner problem [12] is the problem of finding suitable conditions on K such that K is the scalar-curvature for a metric g on S^n conformally equivalent to c. For $n \geqslant 3$, the metric then reads

$$g = u^{4/(n-2)}c,$$

where u is a positive function on S^n satisfying the partial differential equation

$$\text{(P)} \qquad \begin{cases} -Lu = \dfrac{n-2}{4(n-1)} K(x) u^{(n+2)/(n-2)}, \\ u > 0 \quad \text{on } S^n, \end{cases}$$

where $L = \Delta - n(n-2)/4$ is the conformal Laplacian.

In this article, we study problem (P) for $n \geqslant 7$. We show that the ideas introduced in [3], when combined with the ideas introduced in [5], allow us to generalize the results of [3] on the scalar-curvature problem in dimension $n \geqslant 7$.

Theorems 1, 2, 3, and 4, which we prove in this article, considerably extend the topological tools introduced by A. Bahri, and are designed to give to these tools their full strength. Our method still relies strongly on the use of the invariant introduced by A. Bahri in [3], which we extend to derive multiplicity results. However, in this more general situation, where the function K can have several critical points y_i having $-\Delta K(y_i) > 0$, new phenomena appear, related to the fact that the dynamic at infinity is more complicated. The complication is as addressed in [5] and relies on the fact that the flowlines at infinity (i.e., on the unstable manifolds of the critical points at infinity) are not all direct (see the definition below). Also, the dynamic is complicated by several asymptotes.

The indirect flowlines are called *deconcentration connections* (see the definition below). We have to prove that their contribution to the invariant is zero. This follows from the analysis completed in [5].

Despite the multiple references to [3] and [5], our paper is self-complete and the reader does not need to refer to other sources in order to understand it. In some sense, we are completing the program introduced in [3]. In future work,

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