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SPHERICAL VARIETIES AND EXISTENCE OF INVARIANT KÄHLER STRUCTURES

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CONTENTS

1. Introduction. Let K be a compact, connected Lie group acting on projective n-space $\mathbb{P}^n$ via a unitary representation $K \to U(n+1)$. If one gives $\mathbb{P}^n$ a symplectic structure via the Fubini-Study form, then the action of K on $\mathbb{P}^n$ is Hamiltonian, and any smooth K-invariant subvariety $M \subset \mathbb{P}^n$ inherits the structure of a Hamiltonian K-manifold. Naturally one wants to know what class of Hamiltonian actions arises in this way. To phrase a related question, what class of Hamiltonian actions admits an invariant, compatible Kähler structure?

One expects the answers to these questions to depend on the "degree of symmetry" of the symplectic manifold in question. There are many examples of symplectic manifolds without group actions that admit no compatible Kähler structure (see, e.g., [14] and [15]). On the other hand, as observed by Kostant and Souriau, transitive Hamiltonian actions of compact groups are coadjoint orbits and are therefore Kähler. Coadjoint orbits are examples of *multiplicity-free Hamiltonian actions*, which are a class of symplectic manifolds with a very high degree of symmetry. Multiplicity-free torus actions were studied by Delzant [11] (under the name *completely integrable torus actions*), who showed that, under certain assumptions, each of these actions admits an invariant, compatible Kähler structure.

Counterexamples in the nonabelian case were found independently by Knop [30] and the author [45]. In this paper, we consider the question in more detail.

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