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EQUILIBRIUM FLUCTUATIONS FOR THE DISCRETE BOLTZMANN EQUATION

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1. Introduction. A rarefied gas is microscopically modeled by particle systems in which particles travel according to their velocities while gaining new velocities upon collision. As the total number of particles increases unboundedly, the particle density of a given velocity is expected to converge to a solution of a *Boltzmann equation.*

In a *Boltzmann-Grad limit*, the total number of particles N and the range of interaction L^{-1} are related in such a way that NL^{1-d} stays uniformly positive and bounded. Such a scaling constraint results in the uniform positivity and boundedness of the *mean free path* (distance traveled by a particle with no collision) that is responsible for the kinetic behavior of a rarefied gas.

In recent works, Rezakhanlou [R1], [R2] and Rezakhanlou and Tarver [RT] established a kinetic limit for a 1-dimensional model in which NL^{-1} is of order 1 and collision rules are chosen in such a way that when two particles are within a distance L^{-1} they collide with a small probability that is proportional to L^{-1} (see Remark 2.3). In this way, the mean free path is still of order 1 and, as a result, a kinetic behavior is expected.

From a probability point of view, a Boltzmann-Grad limit is a law of large numbers. The main result of this paper is a central limit theorem for the multidimensional version of the model studied in [RT].

The microscopic models studied in this paper are continuous, time Markovian, particle systems with the following rules. Each particle is located on a d-dimensional torus $\mathbb{T}^d$ and has a label α that belongs to the finite set $\{1, 2, \ldots, n\}$. A particle with label α travels deterministically with velocity v_α. Two particles within a distance of order L^{-1} collide stochastically through a continuously differentiable potential V. If two particles of labels α and β collide, they gain new labels γ and δ with a rate $K(\alpha\beta, \gamma\delta)$. If f_α denotes the macroscopic density of particles with label α, then f_α's are expected to satisfy the discrete Boltzmann equation

$$\frac{\partial f_\alpha}{\partial t} + v_\alpha \cdot \frac{\partial f_\alpha}{\partial x} = \sum_{\beta\gamma\delta} K(\gamma\delta, \alpha\beta) f_\gamma f_\delta - K(\alpha\beta, \gamma\delta) f_\alpha f_\beta. \tag{1.1}$$

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