Vol. 93, No. 2 DUKE MATHEMATICAL JOURNAL

TOPOLOGICAL BOUNDARY VALUES AND REGULAR $\mathscr{D}$-MODULES

YVES LAURENT AND TERESA MONTEIRO FERNANDES

To the memory of our friend Emmanuel Andronikof

0. Introduction. Since the development of microlocal analysis in the seventies, several approaches of boundary value problems for hyperfunction and microfunction solutions of complex differential systems were obtained using local, microlocal, or second microlocal tools (see [8], [10], [20], [23], [25]). In [4] some of these results were regarded through the "$\mathscr{D}$-module" point of view, that is, defining the morphism of boundary values as a functor between the hyperfunction solutions of a $\mathscr{D}$-module and the hyperfunction solutions of its vanishing and nearby cycles. These results always assumed a hypothesis of regularity of the Fuchsian type to define the morphism. In [25] Tahara considered an operator simultaneously Fuchsian and hyperbolic and proved that the morphism of boundary value is an isomorphism in this case.

In this paper, our first aim is to prove a result analogous to Tahara's in the framework of a $\mathscr{D}$-module of [4]. Working with a $\mathscr{D}$-module, we cannot assume that an operator is simultaneously Fuchsian and hyperbolic. Therefore, we have to separate the hypotheses and prove that if the module is Fuchsian (more precisely, regular along a hypersurface), then there is a boundary value morphism, and if we add a hypothesis of hyperbolicity, then it is an isomorphism.

In fact, it appears that the condition of regularity itself is not necessary. If we assume a geometrical condition on the characteristic variety of the $\mathscr{D}$-module, we can define a boundary value morphism, which is an isomorphism with the condition of hyperbolicity (which itself is a geometric condition on the characteristic variety). In this context, the morphism has values in the geometric vanishing and nearby cycles of the sheaf of solutions of $\mathscr{M}$. Now the regularity condition is used only to identify these geometric vanishing cycles of solutions with solutions of a $\mathscr{D}$-module, called the vanishing cycles of the $\mathscr{D}$-module.

Once these conditions have been reduced to conditions on the characteristic variety of the $\mathscr{D}$-module, it appears that we may forget completely $\mathscr{D}$-modules and use the theory of sheaves of Kashiwara-Schapira [13]. More precisely, we prove a theorem for the complex of sheaves of $\mathbb{C}$-vector spaces, the conditions

Received 27 December 1996.
1991 *Mathematics Subject Classification.* Primary 35A27, 58G7, 58G20.
Monteiro Fernandes's work supported by Agreement Research JNICT/Centre National de la Recherche Scientifique/1994 and European Community Contract CHRX-CT 94-0609.