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NIEMEIER LATTICES, MATHIEU GROUPS, AND FINITE GROUPS OF SYMPLECTIC AUTOMORPHISMS OF $K3$ SURFACES

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APPENDIX BY SHIGERU MUKAI

§0. Introduction. Let X be a $K3$ surface; that is, X is a simply connected, compact, complex surface with a trivial canonical bundle. An automorphism g of X is called *symplectic* if g acts identically on a nowhere-vanishing holomorphic 2-form on X. In [6], Nikulin studied finite automorphisms of $K3$ surfaces and gave a classification of finite abelian groups of symplectic automorphisms of $K3$ surfaces. Mukai [4] observed that for any symplectic automorphism g of a $K3$ surface, the number of fixed points of g is equal to that of an element of the same order in the Mathieu group M_{23} acting on a set Ω of twenty-four elements. Based on this fact, he determined eleven maximal groups, each of which can act symplectically on a $K3$ surface. These groups are characterized as maximal groups, which are isomorphic to subgroups in M_{23} with at least five orbits on Ω. However, the proof of this result is long and contains rather difficult discussions.

The purpose of this note is to provide a simple proof of the following theorem.

THEOREM [4, Thm. 0.3]. *Let G be a finite group of symplectic automorphisms of a $K3$ surface. Then G is isomorphic to a subgroup of the Mathieu group M_{23}, which has at least five orbits on Ω.*

We give a sketch of our proof of this theorem. Let X be a $K3$ surface and let G be a finite group acting on X as symplectic automorphisms. Let L_G be the sublattice of $H^2(X,\mathbf{Z})$, which is the orthogonal complement to the invariant sublattice $H^2(X,\mathbf{Z})^G$. It is known (see [6]) that L_G is an even, negative, definite lattice with rank less than or equal to 19, and that it contains no (-2)-elements. We shall see that L_G can be primitively embedded into an even, negative, definite, unimodular lattice N of rank 24 with a (-2)-element. G can also be considered as a subgroup of the symmetry group of the Dynkin diagram of N that fixes at least one vertex. (Note that the number of the vertices of the Dynkin diagram of N is twenty-four.) This implies that G can be embedded into the Mathieu group M_{23} with at least five orbits.

The proof of the above theorem is given in §3. Our proof is a purely lattice-theoretic one, based on the following results: Nikulin's theory of embeddings of even lattices into even unimodular lattices (see [7]); the classification of even,

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