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ANALYTIC TRANSFORMATIONS OF $(\mathbf{C}^p, 0)$ TANGENT TO THE IDENTITY

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1. Introduction. Let F be a germ of analytic transformation from $(\mathbf{C}^p, 0)$ to $(\mathbf{C}^p, 0)$ tangent to the identity, that is, given by convergent power series in a neighborhood of zero as

$$X_1 = X + P_2(X) + P_3(X) + \cdots, \tag{1.1}$$

where, for $h \in \mathbf{N}$, P_h is a homogeneous polynomial function of degree h from $\mathbf{C}^p$ to $\mathbf{C}^p$. We are interested in the behavior of the iterates $X_n = F^{(n)}(X)$ of points near the origin. Our aim is to find invariant manifolds or invariant domains attracted by the origin.

Such transformations in $(\mathbf{C}, 0)$ were first studied by Fatou and Leau [F1], [L]. Their theory is quite complete (for a modern exposition see [Be] or [CG]).

In his paper on the transformations of $(\mathbf{C}^2, 0)$, [F2], Fatou raised the problem of describing the behavior of transformations of $(\mathbf{C}^2, 0)$ tangent to the identity. He considered this problem as difficult, and he dealt only with the particular case of maps of the type

$$\begin{cases} x_1 = \dfrac{x}{1 + xP(x,y)}, \\ y_1 = \dfrac{y}{1 + yQ(x,y)} \end{cases} \tag{1.2}$$

with $P(0,0), Q(0,0) \neq 0$. It is interesting to notice that in this example, the transformation has two invariant analytic curves through the origin. As we shall see, that is very exceptional.

Since then, a complete classification of the analytic transformations of $(\mathbf{C}^p, 0)$, tangent to the identity, is given by Ecalle in [E]. His results are a consequence of his theory of resurgent functions, which is, by itself, a very deep and elaborate work.

Let $\{\varphi^t\}$ be the flow of a differential system in $(\mathbf{C}^p, 0)$, defined by a convergent power series of order 2 at the origin

$$X' = P_2(X) + \tilde{P}_3(X) + \cdots. \tag{1.3}$$

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