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EVOLVING MONOTONE DIFFERENCE OPERATORS ON GENERAL SPACE-TIME MESHES

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1. Introduction. In previous papers [10] and [11] we considered positive and monotone difference operators on general meshes in Euclidean space that corresponded to elliptic partial differential operators. The methods and results in these papers extended our original work for cubic meshes [6]. In this paper, we treat monotone difference operators on general meshes in space-time that correspond to parabolic partial differential operators, thereby extending our previous work [8] and [9] for spatially cubic meshes. In particular, we prove a general discrete analogue of the Krylov maximum principle [4] under reasonably optimal hypotheses (Theorem 2.1). Using this result, we prove a local maximum principle, Theorem 3.1, which is also significant in that the restriction to implicit schemes in the spatially cubic case of [9] is removed through our more general techniques. For the other local estimates, notably the discrete Harnack and Hölder estimates, presented in Theorems 4.2, 4.3, and 4.4, we consider non-explicit schemes, and their derivation is essentially an amalgamation of our ideas in [9] and [10].

Let us first recall some definitions from [9] and [10]. For an arbitrary set E, called a *mesh*, a linear difference operator acting on $\mathfrak{M}(E)$, the space of real mesh functions, is given by

$$Lu(x) = \sum_{y \in E} a(x, y)u(y) \tag{1.1}$$

for any mesh function u, where a is a real-valued function on $E \times E$, which is nonvanishing for only a finite number of y values for each $x \in E$. The operator L is of *monotone type* if

$$a(x, y) \geqslant 0, \qquad \text{for all } (x, y) \in E \times E, \quad x \neq y, \tag{1.2}$$

and of *positive type* if, in addition,

$$c(x) \equiv \sum_{y} a(x, y) \leqslant 0. \tag{1.3}$$

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