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DISCRETE CONVOLUTION-REARRANGEMENT INEQUALITIES AND THE FABER-KRAHN INEQUALITY ON REGULAR TREES

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CONTENTS

1. Introduction. Recall the classical Riesz convolution-rearrangement inequality in [24] (see [9] for a more general result)

$$\int_{\mathbb{R}}\int_{\mathbb{R}} f(x)g(x-y)h(y)\,dxdy \leqslant \int_{\mathbb{R}}\int_{\mathbb{R}} f^{\#}(x)g^{\#}(x-y)h^{\#}(y)\,dxdy$$

for positive functions on $\mathbb{R}$, where, given a function F on $\mathbb{R}$, the function $F^{\#}$ is the *symmetric decreasing rearrangement* of F. That is, the unique lower-semicontinuous function on $\mathbb{R}$, which is equimeasurable with F, is (not necessarily strictly) decreasing on $[0,\infty)$, and satisfies $F^{\#}(x) = F^{\#}(-x)$ for all x. Of special interest is the case where g is already symmetric decreasing, that is, where $g^{\#} = g$. That case can be expressed as

$$\int_{\mathbb{R}}\int_{\mathbb{R}} f(x)K(|x-y|)h(y)\,dxdy \leqslant \int_{\mathbb{R}}\int_{\mathbb{R}} f^{\#}(x)K(|x-y|)h^{\#}(y)\,dxdy \tag{1.1}$$

for a decreasing function K on $[0,\infty)$. Inequalities like (1.1) are valid for a number of different types of rearrangements on $\mathbb{R}^n$, $\mathbb{S}^n$, and $\mathbb{H}^n$ and often have inter-

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