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TIME DECAY FOR THE BOUNDED MEAN OSCILLATION OF SOLUTIONS OF THE SCHRÖDINGER AND WAVE EQUATIONS

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1. Introduction. Consider the wave equation

$$\partial_t^2 u(t,x) = \Delta u(t,x),$$

$$u(0,x) = 0,$$

$$\partial_t u(0,x) = f,$$

where $t \in \mathbf{R}$, $x \in \mathbf{R}^d$, and $f \in L_2(\mathbf{R}^d)$. Let us write once and for all $B_t f = u(t,\cdot)$, so that $\widehat{B_t f}(\zeta) = (\sin(t|\zeta|)/|\zeta|)\hat{f}(\zeta)$. In this paper we discuss the endpoints of the following assertion.

Assertion $(B_{d,p,q})$. *If $f \in L_2(\mathbf{R}^d)$, then $t \mapsto B_t f$ is in $L_q(\mathbf{R}, L_p(\mathbf{R}^d))$, and there exists a constant c independent of f such that*

$$\left(\int_{-\infty}^{\infty} \|B_t f\|_p^q \, dt\right)^{1/q} \leqslant c\|f_2\|.$$

If $p, q \geqslant 1$, then by standard arguments, it is easy to show that this is equivalent to its dual assertion. Here, as in the rest of the paper, $1/p + 1/p' = 1/q + 1/q' = 1$.

Assertion $(B^*_{d,p',q'})$. *If $t \mapsto f_t$ is in $L_{q'}(\mathbf{R}, L_{p'}(\mathbf{R}^d))$, then $\int_{-\infty}^{\infty} B_t f_t \, dt$ (exists almost everywhere and) is in $L_2(\mathbf{R}^d)$, and there exists a constant c independent of f_t such that*

$$\left\|\int_{-\infty}^{\infty} B_t f_t \, dt\right\|_2 \leqslant c\left(\int_{-\infty}^{\infty} \|f_t\|_{p'}^{q'} \, dt\right)^{1/q'}.$$

In stating these assertions, we always suppose that d, p, and q satisfy the fol-

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