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A BOCHNER THEOREM AND APPLICATIONS

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0. Introduction. We begin by recalling a *pure F-structure* and relevant related notions; for details see [CG1], [CG2], [CR1], [CR2].

Let M be a manifold. $\mathscr{F}$, a pure F-structure on M, is a kind of generalized torus action on M. By definition, a pure F-structure is a *flat torus bundle* over M with *holonomy* in the automorphism group of the torus and its local *action* on M. With a choice of a base point, the holonomy group is a subgroup of $SL(s, \mathbb{Z})$, where s is the rank of the torus. The action is a homomorphism from the associated bundle of Lie algebra to the sheaf of local smooth vector fields over M. A subset of M is called *invariant* if it is preserved by the infinitesimals of the local fields, which are the homomorphic images of the associated bundle of Lie algebra. An *orbit* is a smallest invariant subset. The rank of $\mathscr{F}$ is the dimension of an orbit of smallest dimension. A metric is called *invariant* if the homomorphic images are local Killing fields. For a pure F-structure, there always is an invariant metric.

A global torus action defines a pure F-structure. However, a pure F-structure with a nontrivial holonomy group is not defined by a global torus action. Moreover, manifolds that only admit the trivial torus action may admit nontrivial pure F-structures. (For example, there are 3-dimensional solvable manifolds that admit only the trivial torus action and that admit only nontrivial F-structures.)

The main result in this paper is the following.

THEOREM 0.1. *A compact manifold of negative Ricci curvature does not admit any nontrivial invariant pure F-structure.*

Since a nontrivial Killing field (on a compact manifold) implies a nontrivial isometric torus action (i.e., the closure of the 1-parameter subgroup of the Killing field), Theorem 0.1 implies the following classical Bochner theorem.

COROLLARY 0.2. *A compact manifold of negative Ricci curvature does not admit any nontrivial Killing field.*

Note that, in comparison with the classical Bochner theorem, Theorem 0.1 provides us with a *constraint on local geometric structure* of a metric with negative Ricci curvature, since a nontrivial invariant pure F-structure is a kind of a local symmetric structure. This constraint will have interesting applications in

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