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ON p-ADIC CONVERGENCE OF PERTURBATIVE INVARIANTS OF SOME RATIONAL HOMOLOGY SPHERES

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1. Introduction. The cyclotomic properties of the $SO(3)$ Witten-Reshetikhin-Turaev (WRT) invariant $Z'(M;K)$ of $3d$-manifolds M have attracted a lot of attention recently. This invariant was defined by R. Kirby and P. Melvin [7] by modifying the Reshetikhin-Turaev surgery formula of [18]. H. Murakami showed that if M is a rational homology sphere (RHS) and K is an odd prime number, then

$$Z'(M;K) \in \mathbb{Z}[\check{q}], \qquad \check{q} = e^{(2\pi i)/K}. \tag{1.1}$$

Here $\mathbb{Z}[\check{q}]$ is a cyclotomic ring

$$\frac{\check{q}^K - 1}{\check{q} - 1} = 0. \tag{1.2}$$

(Alternative proofs of (1.1) were presented in [12] and [23].)

As an element of $\mathbb{Z}[\check{q}]$, $Z'(M;K)$ can be presented as a polynomial in $\check{q} - 1$ as follows:

$$Z'(M;K) = \sum_{n=0}^{K-2} a_n(M;K)h^n, \qquad h = \check{q} - 1. \tag{1.3}$$

The numbers $a_n(M;K) \in \mathbb{Z}$ depend on both the rational homology sphere M and the "level" K. T. Ohtsuki showed in [16] and [17] how to reprocess $a_n(M;K)$ into the invariants of M which are independent of K.

We introduce the following notation. Since our prime number is K, we use the term "K-adic" instead of the usual term p-adic. For $q \in \mathbb{Z}, q \not\equiv 0 \pmod{K}$, let q^* denote the K-adic inverse of q. In other words, q^* is a formal series in positive powers of K

$$q^* = \sum_{n=0}^{\infty} q^*_{(n)} K^n, \qquad 0 \leqslant q^*_{(n)} \leqslant K - 1, \tag{1.4}$$

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