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QUANTITATIVE UNIQUENESS FOR SECOND-ORDER ELLIPTIC OPERATORS

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1. Introduction. In [DF1] and [DF2], Donnelly and Fefferman proved that if u is an eigenfunction on a compact smooth Riemannian manifold M (subject to Dirichlet or Neumann boundary conditions) (i.e., if $u \not\equiv 0$ and

$$-\Delta_M u = \lambda u$$

for some $\lambda > 0$), then the maximal order of vanishing of u on M is less than $\mathrm{const} \cdot \lambda^{1/2}$. This method is based on certain Carleman estimates for the operator $\Delta + \lambda$, and it does not seem to extend to more general equations of the form

$$-\Delta_M u = V(x)u, \tag{1.1}$$

where $V \in L^\infty(M)$.

The purpose of the present paper is to consider the more general case (1.1). Namely, in Theorem 5.2, we prove that the maximal order of vanishing for $u \not\equiv 0$, which solves (1.1), is less than

$$\mathrm{const}\left(1 + (\sup_M V_-)^{1/2} + (\operatorname*{osc}_M V)^2\right), \tag{1.2}$$

where the constant depends only on the geometry of M. This upper bound clearly agrees with the upper bound of Donnelly and Fefferman in the eigenvalue case $V \equiv \lambda$.

In mathematical literature, there are essentially two different methods for showing strong unique continuation for the equations where V is not analytic. One involves Carleman-type estimates (see [K], [JK], [DF1], [L], [J], and [S], to name only a few references), and the other is the use of the frequency function of Garofalo and Lin [GL] (see also [A], [B], and [KM]). Both methods study the behavior of solutions in sufficiently small balls (the size of the ball depends on the size of V). Following the dependence on V in these methods, one thus obtains bounds that are not algebraic in $\|V\|_{L^\infty}$. Using the transformation

$$\tilde{u}(x,y) = u(x)\cosh(\lambda y), \qquad (x,y) \in M \times [-1,1] \tag{1.3}$$

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