Vol. 91, No. 1 DUKE MATHEMATICAL JOURNAL

GROUP COHOMOLOGY AND THE SYMPLECTIC STRUCTURE ON THE MODULI SPACE OF REPRESENTATIONS

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0. Introduction. The space of equivalence classes of irreducible representations of the fundamental group of a compact-oriented surface of genus at least 2 in a Lie group has a natural symplectic form. In [AB], Atiyah and Bott described this symplectic structure using methods from gauge theory. Goldman [G] constructed the skew-symmetric pairing algebraically using methods from group cohomology. Using Poincaré duality, Goldman showed that the pairing is nondegenerate and identified it with the symplectic structure given by gauge theory.

For a punctured surface, the space of equivalence classes of irreducible representations also admits a symplectic structure, provided some boundary conditions are imposed (namely, fixing the conjugacy classes of the holonomies around the punctures). The symplectic structure on the moduli space was constructed analytically using gauge theory in [BG1] and [BG2]. An algebraic description of the symplectic form using parabolic cohomology is given in [GHJW]. The required nondegeneracy is proved using the cohomology of *group systems*; this can be thought of as an analogue of compactly supported cohomology.

This paper can be considered a companion to [GHJW]. The principal aim of this paper is to prove, using ideas from the Hodge theory of forms and the explicit use of Fox calculus, the nondegeneracy of the skew-symmetric pairings which then give the above symplectic structures.

In the process of the proof, we introduce a Riemannian metric on the above spaces of equivalence classes of representations of the fundamental group. The choice of this metric was motivated and in some sense dictated by the explicit description of the symplectic structure by Goldman [G] in the compact case. We came up with this metric in an attempt to mimic a harmonic theory of forms in the cohomology of groups. This metric, we believe, is interesting in its own right and needs further study. The striking feature of this metric on the moduli space of representations is that it can be given explicitly once a presentation of the fundamental group of the surface is chosen and does not apparently decode any complex structure or metric on the surface itself.

In the first section we describe the metric on the moduli space in the compact surface case and prove the nondegeneracy of the skew-symmetric pairing, which

Received 25 August 1995. Revision received 31 October 1996.