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SERRIN'S RESULT FOR HYPERBOLIC SPACE AND SPHERE

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§1. Introduction. In [5] "A symmetry problem in potential theory," J. Serrin proved that for a bounded domain $\Omega \subset \mathbb{R}^n$, a positive solution of the differential equation

$$\Delta u + f(u) = 0 \quad \text{in } \Omega \tag{1}$$

with boundary conditions

$$u = 0, \qquad \frac{\partial u}{\partial \nu} = \mathit{constant} \quad \text{on } \partial\Omega, \tag{2}$$

where f is a C^1 function, is radially symmetric, and that Ω is a ball. In this paper, we prove a similar result for bounded domains in the n-dimensional hyperbolic space $\mathbf{H}^n$ and sphere $\mathbf{S}^n$. We shall consider a bounded domain Ω with C^1 boundary and equations (1) and (2), where Δ is the Laplace-Beltrami operator of respective spaces. More precisely, we prove the following theorem.

THEOREM 1. *Let $\Omega \subset \mathbf{H}^n$ be a bounded domain. Let $u \in C^2(\bar{\Omega})$ be a positive solution of*

$$\Delta u + f(u) = 0 \quad \mathit{in}\ \Omega \tag{3}$$

with boundary conditions

$$u = 0, \qquad \frac{\partial u}{\partial \nu} = c \quad \mathit{on}\ \partial\Omega, \tag{4}$$

where f is a C^1 function, c is a constant, and $\frac{\partial u}{\partial \nu}$ denotes the normal derivative along the inward normal field on $\partial\Omega$. Then

(i) *Ω is a geodesic ball, and*
(ii) *u is radially symmetric.*

In [5], Serrin used the method of moving planes to prove the symmetry of the solution as well as the domain. The moving plane method works for domains in Euclidean space $\mathbb{R}^n$ since the isometries of $\mathbb{R}^n$ are generated by reflections with

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