Vol. 90, No. 3 DUKE MATHEMATICAL JOURNAL © 1997

METAPLECTIC EISENSTEIN SERIES AND THE BUMP-HOFFSTEIN CONJECTURE

TOSHIAKI SUZUKI

Introduction. By *a metaplectic form* we understand an automorphic function on an n-fold cover of GL_r over a global field $\mathbf{k}$ containing nth roots of unity. According to Kubota, the *theta series* Θ_r is defined to be a metaplectic form that is obtained as a residue of metaplectic Eisenstein series. We are chiefly interested in understanding the Fourier coefficient $\mathbf{a}_r$ of Θ_r. In [KP] we find the first fundamental result (as well as the formulation and the terminology) in the theory of metaplectic forms, from which some information on $\mathbf{a}_r$ can be drawn: in particular, $\mathbf{a}_r = 0$ for $r > n$, and $\mathbf{a}_n$ is expressed as a product of nth-order Gauss sums. The author's result, obtained in [S1], also seems to be fundamental: a duality between $\mathbf{a}_r$ and $\overline{\mathbf{a}}_{n-r}$ (the complex conjugation of $\mathbf{a}_{n-r}$) with respect to $\overline{\mathbf{a}}_n$ (see Theorem 8.4). In fact, we could partly prove Patterson's conjecture on the biquadratic theta series via this relation (see [S1], [S2]).

In this paper, we show that Bump-Hoffstein's conjecture (see [BH1]) is also a consequence of our result. This conjecture states a relationship between

(1) the Fourier coefficients of the Eisenstein series $E(s, \Theta_{r_1} \otimes \Theta_{r_2})$ associated to a tensor product of theta series $\Theta_{r_1} \otimes \Theta_{r_2}$ ($r_1, r_2 \leqslant n$), and
(2) the two Rankin-Selberg convolutions for pairs of theta series $(\Theta_{r_1}, \overline{\Theta}_{n-r_2})$ and $(\Theta_{r_2}, \overline{\Theta}_{n-r_1})$.

For the reason stated in [S1], we shall prove the case where the ground field $\mathbf{k}$ is a function field. However, we shall not assume that $\mathbf{k}$ is a function field until Section 7. This is because we want to mention that a Dirichlet series whose coefficients are nth power Legendre symbols

$$\sum_{\ell} \varepsilon \left(\frac{m}{\ell}\right)_S^{-1} |\ell|_\infty^{-s-1}$$

will appear as a Fourier coefficient of $E(s, \Theta_1 \otimes \Theta_{n-1})$ (Proposition 6.4). Bump and Hoffstein observed this fact in the case $n = 3$ [BH2].

We freely make use of the results from [KP]. In Part I (local theory), after recalling the notation that we need, we compute some class-1 Whittaker functions at certain points using Patterson's formula [P]. This result is of some independent interest. In Part II (global theory), we start with introducing the Eisenstein series $E(s, \Theta_{r_1} \otimes \Theta_{r_2})$. In Section 6, by the local result mentioned above, we show that a certain Fourier coefficient $\Psi(s, \Theta_{r_1} \otimes \Theta_{r_2})$ of $E(s, \Theta_{r_1} \otimes \Theta_{r_2})$ is equal to a

Received 12 August 1996.