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STRUCTURE OF THE RESOLVENT FOR THREE-BODY POTENTIALS

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1. Introduction. In this paper we construct a parametrix for the resolvent of the three-body Hamiltonian $H_V = \Delta + \sum_i V_i$ when the two-body potentials are real valued and Schwartz: $V_i \in \mathscr{S}(X^i)$. Here Δ is the positive Laplacian on $\mathbb{R}^N$, and X^i are linear subspaces of $\mathbb{R}^N$. The (reduced) three-body problem has the important property that $X_i \cap X_j = \{0\}$ for $i \neq j$ where $X_i = (X^i)^\perp$. The construction is performed via a finite iteration of the two-body resolvents which is in many ways similar to Faddeev's original approach [10]. This is facilitated by the use of the scattering calculus introduced by Melrose in [29] and motivated by the parametrix construction of Melrose and Zworski for the Poisson operator in [31].

Thus, we identify $\mathbb{R}^N$ with the interior of its radial compactification $\mathbb{S}^N_+$, the upper hemisphere, via the stereographic projection $\mathrm{SP}\colon \mathbb{R}^N \to \mathbb{S}^N_+$. Let $C_i = \bar{X}_i \cap \mathbb{S}^{N-1}$ where $\mathbb{S}^{N-1} = \partial \mathbb{S}^N_+$, $\bar{X}_i = \mathrm{cl}(\mathrm{SP}(X_i))$. Then $V_i \in (\mathbb{S}^N_+ \backslash C_i)$ and vanishes to infinite order on $\mathbb{S}^{N-1} \backslash C_i$. Note that the condition $X_i \cap X_j = \{0\}$ for $i \neq j$ becomes $C_i \cap C_j = \varnothing$. Our approach will enable us to describe the asymptotic behavior of the resolvent applied to Schwartz functions and to obtain the structure of the three-cluster to three-cluster part of the scattering matrix. Namely, we prove the following theorem, which was conjectured by Melrose (motivated by [31]).

THEOREM. *The three-cluster to three-cluster part of the (absolute) scattering matrix is a sum of Fourier integral operators on* $\mathbb{S}^{N-1} \backslash \bigcup_i C_i$ *associated to the "broken" geodesic flow, broken at points in* $\bigcup_i C_i$, *at distance* π.

Andrew Hassell had previously proved the corresponding theorem for the scattering matrix of the unreduced two-body Hamiltonian (i.e., the case when we only have one two-body potential) [16].

We denote the (modified) resolvent of H_V by $R_V(\lambda)$. Thus, our normalization is that of [30], so $R_V(\lambda) = (H_V - \lambda^2)^{-1}$ in the "physical half-plane" where $\operatorname{Im} \lambda < 0$, and for $\sigma > 0$ $(H_V - (\sigma \pm i0))^{-1} = R_V(\mp \sigma^{1/2})$. We also show in this paper that away from $\bigcup_i C_i$, $R_V(\lambda) f$ (here $f \in \mathscr{S}(\mathbb{R}^N)$) has the same behavior as in the free problem; that is, for $\lambda > 0$

$$R_V(\lambda) f(r\theta) \sim e^{-i\lambda r} r^{-(N-1)/2} \sum_{j \geq 0} r^{-j} g_j(\theta)$$

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