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TWISTED SYMMETRIC-SQUARE L-FUNCTIONS AND THE NONEXISTENCE OF SIEGEL ZEROS ON $GL(3)$

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§1. Introduction. In a lecture given at the Workshop on Automorphic Forms at the Mathematical Sciences Research Institute in October 1994, D. Goldfeld emphasized the importance of establishing the conjecture that the standard L-function of a cuspidal automorphic representation π on $GL(n)$ does not, when $n > 1$, admit a *Siegel zero*, i.e., a real zero of $L(s, \pi)$ that lies close to $s = 1$. This is also conjectured for $n = 1$, although the problem is much deeper in that case. For $n = 2$, the Siegel zero conjecture was proved unconditionally by Hoffstein and Ramakrishnan [5]. They were also able to establish the result when $n = 3$ under the assumption of a certain hypothesis [5, Hypothesis 6.1]. The main result of this paper is the proof of that hypothesis, which thus establishes the Siegel zero conjecture unconditionally for $n = 3$.

Let $\mathbb{F}$ be an algebraic number field, with $\mathbb{A}$ its ring of adèles, and let π be a unitary cuspidal automorphic representation of $GL(3, \mathbb{A})$. In this paper, we will prove the following.

THEOREM 1. *$L(s, \pi)$ does not admit a Siegel zero.*

THEOREM 2. *Let ω be the central character of π, and let χ be a Hecke character. Then the partial Langlands L-function $L_S(s, \pi, \bigvee^2 \otimes \chi)$ (defined below) extends to a meromorphic function of $s \in \mathbb{C}$, with possible simple poles occurring at $s = 0$ and $s = 1$. There is no pole unless $\chi^3 \omega^2 = 1$.*

Theorem 2 is a restatement of Theorem 7, which is proved below in §3. An immediate corollary to Theorem 2 is the proof of Hypothesis 6.1 of [5], which implies Theorem 1. Although the theorems above are new, the techniques of proof are entirely contained in the papers listed as references, most notably [1], [2], [5], [6], and [9].

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§2. The nonexistence of Siegel zeros on $GL(3)$. Let $\mathbb{F}$ be an algebraic number field, with $\mathbb{A} = \mathbb{A}_{\mathbb{F}}$ its ring of adèles. For every integer $n \geqslant 1$, let $\mathscr{A}_n^\circ(\mathbb{F})$ denote

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