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THE VARIATIONAL BICOMPLEX FOR HYPERBOLIC SECOND-ORDER SCALAR PARTIAL DIFFERENTIAL EQUATIONS IN THE PLANE

IAN M. ANDERSON AND NIKY KAMRAN

§1. Introduction. Over the past two decades, there has been a great resurgence in the differential geometric investigation of partial differential equations, a subject pioneered and quite extensively developed around the turn of the century by Lie, Darboux, Goursat, Janet, E. Cartan, Vessiot, and others. Topics of interest today include the systematic computation of (generalized) symmetries and conservation laws, classical and nonclassical methods of reduction, Hamiltonian and bi-Hamiltonian structures, the inverse problem of the calculus of variations, Bäcklund transformations, Painlevé properties, and equivalence problems. Much of this activity is directed towards the goal of achieving a better understanding of the phenomena of complete integrability for partial differential equations. The purpose of this paper is to carry out a systematic study of some of these issues for second-order hyperbolic scalar partial differential equations

$$F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}) = 0, \tag{1.1}$$

in two independent variables x and y and one dependent variable u. We shall focus, in particular, on the calculation of conservation laws.

A *classical conservation law* for the partial differential equation (1.1) is a one-form

$$\omega = M\,dx + N\,dy,$$

where M and N are functions of x, y, u, and the derivatives of u up to finite order, such that

$$D\omega = (D_xN - D_yM)\,dx \wedge dy = 0$$

by virtue of the equation (1.1) and its differential consequences. Here D_x and D_y denote the total derivatives with respect to the independent variables x and y, and we have used the symbol D to emphasize that the exterior derivative of the

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