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LOCAL SOLVABILITY OF ANALYTIC PSEUDODIFFERENTIAL COMPLEXES IN TOP DEGREE

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1. Introduction. The present work concerns the local solvability of analytic pseudodifferential complexes in top degree associated to a Frobenius ideal of codimension one in the ring of pseudodifferential operators. Without further specification, all operators are analytic throughout this paper. Let Ω be an open subset in $\mathbb{R}^{n+1}$ and $\Psi(\Omega)$ be the ring of pseudodifferential operators on Ω. A left ideal $\mathscr{P}$ in $\Psi(\Omega)$ is said to be a Frobenius ideal of rank n if, for every point $y \in \Omega$, there exists an open neighborhood U of y and n first-order pseudodifferential operators $P_1, \dots, P_n \in \mathscr{P}$, such that $P_1, \dots, P_n$ are linearly independent over $\Psi(U)$, and for all $P \in \mathscr{P}$, there exist $A_1, \dots, A_n \in \Psi(U)$ such that

$$P = \sum_{i=1}^{n} A_i P_i,$$

and there exist $c_{jk}^l \in \Psi^0(\Omega)$ such that

$$[P_j, P_k] = \sum_l c_{jk}^l P_l.$$

We define the characteristic set of $\mathscr{P}$ to be $\operatorname{char} \mathscr{P} = \operatorname{char} P_1 \cap \cdots \cap \operatorname{char} P_n$. Our entire analysis will focus on a neighborhood of 0. $\mathscr{P}$ is said to be elliptic at 0 if $\operatorname{char} \mathscr{P}|_0 = \{0\}$. In this paper, we assume that $\mathscr{P}$ is nonelliptic at 0. Furthermore, we require that $\mathscr{P}$ has *simple real* characteristics, that is, for every $\gamma = (y, \eta) \in \operatorname{char} \mathscr{P}$, $d_\eta p_{1,0}, \dots, d_\eta p_{n,0}$ are independent over $\mathbb{C}$ at γ, where η is the fiber variable of the cotangent bundle $T^*\Omega$, and $p_{1,0}, \dots, p_{n,0}$ are the *principal symbols* of $P_1, \dots, P_n$, respectively. We study the local solvability of the following equation:

$$P_1 u_1 + P_2 u_2 + \cdots + P_n u_n = f. \tag{1.1}$$

Equation (1.1) is *locally solvable* at 0 if there exist open neighborhoods $V \subset U$ of 0, such that for any function $f \in C^\infty(U)$, there exist $u_1, u_2, \dots, u_n \in \mathscr{E}'(U)$ which satisfy (1.1) in V. For convenience, usually we write (1.1) as

$$Pu = f,$$

where $P = (P_1, P_2, \dots, P_n)$, $u = (u_1, u_2, \dots, u_n)^T$, T denotes the transpose.

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