Vol. 86, No. 3 DUKE MATHEMATICAL JOURNAL

AN INTEGRAL TRANSFORM FOR p-ADIC SYMMETRIC SPACES

PETER SCHNEIDER AND JEREMY TEITELBAUM

Introduction. One of the important tools in the study of the arithmetic of Shimura curves and other modular curves over a local field K is the theory of p-adic uniformization. Applying earlier work of Ihara and Mumford's general theory of uniformization of certain types of curves over K [Mu], Cerednik [C] showed that one could obtain certain Shimura curves as the quotient of a certain rigid analytic space, the "p-adic upper half-plane," by appropriate cocompact arithmetic subgroups of $\mathrm{GL}_2(K)$. The p-adic upper half-plane of this theory is the rigid analytic space over K obtained by deleting the K-rational points from the projective line over K.

Drinfeld [D] reconsidered Cerednik's results, working with the more general p-adic symmetric spaces $\Omega^{(d+1)}$ obtained by deleting all K-rational hyperplanes from projective d-space over K. He showed that $\Omega^{(d+1)}$ is a moduli space for a certain type of formal group and constructed a tower of etale coverings of $\Omega^{(d+1)}$; these coverings are the topic of considerable interest, in part because of Drinfeld's hope that their cohomology realizes all of the discrete series representations of $\mathrm{GL}_{d+1}(K)$.

In 1991, [SS] studied the cohomology of the spaces $\Omega^{(d+1)}$ in any cohomology theory which satisfied certain natural axioms. In particular, if the field K has characteristic zero, they determined the de Rham cohomology of the spaces $\Omega^{(d+1)}$ and interpreted the results in representation-theoretic terms.

Applied to the one-dimensional p-adic upper half-plane, the results of [SS] give an abstract isomorphism

$$\mathrm{H}^1_{DR}(\Omega^{(2)}, K) \to \mathrm{Hom}(C^\infty(\mathbb{P}^1(K), \mathbb{Z})/\text{constants}, K),$$

where $C^\infty(\mathbb{P}^1(K), \mathbb{Z})$ is the space of locally constant functions on $\mathbb{P}^1(K)$. A number of versions of this result were known prior to the work in [SS]; this prior work relied on analytic techniques available in the one-dimensional case. Two of these techniques were, first, the explicit "residue" map of [S1]:

$$\mathrm{Res}\colon \Omega^1_K \to \mathrm{Hom}(C^\infty(\mathbb{P}^1(K), \mathbb{Z})/K, K),$$

Received 6 December 1994. Revision received 20 March 1996.

Teitelbaum partially supported by the National Science Foundation and by a Sloan Research Fellowship.