Vol. 85, No. 2 DUKE MATHEMATICAL JOURNAL © November 1996

# CHOW RINGS OF INFINITE SYMMETRIC PRODUCTS

SHUN-ICHI KIMURA AND ANGELO VISTOLI

**Introduction.** Let $A$ be an Abelian variety, $N$ an integer with $N \geqslant 2$, $N_A: A \to A$ the morphism given by multiplication by $N$, and $N^* = N_A^*: \mathrm{CH}^*(A) \to \mathrm{CH}^*(A)$ the pullback on the Chow ring of $A$. Bloch [Bl] proved that if $A$ is an Abelian surface, then $N^*$ acts like multiplication by $N^2$ on the kernel of the albanese map $\mathrm{CH}_0(A) \to \mathrm{Alb}(A)$. In 1986, translating $K$-theoretic results by Mukai [Muk] to the language of Chow groups, Beauville [Be1], [Be2] found the following results:

(i) The $p$-codimensional Chow group (with rational coefficients) of an Abelian variety $X$ decomposes into finite direct sum $\oplus \mathrm{CH}_s^p(X)$, where $\mathrm{CH}_s^p(X)$ is the eigenspace of the action by the multiplication by $N$ with the eigenvalue $N^{2p-s}$.

(ii) Let $\ell$ be the first Chern class of the Poincaré line bundle on $X \times \hat{X}$; then the correspondence

$$\mathscr{F} = \exp(\ell) = \sum_{k=0}^{\infty} \frac{\ell^k}{k!}: X \vdash \hat{X}$$

induces a bijection $\mathscr{F}_*: \mathrm{CH}(X) \to \mathrm{CH}(\hat{X})$. The bijection $\mathscr{F}_*$ sends the eigenvalue decomposition to dimension decomposition; more precisely, when $\alpha \in \mathrm{CH}^p(X)$, then $\alpha \in \mathrm{CH}_s^p(X)$ if and only if $\mathscr{F}_*(\alpha) \in \mathrm{CH}^{g-p+s}(\hat{X})$ where $g = \dim(X)$. It also sends Pontryagin product to intersection product, and $N_*$ to $N^*$. The correspondence $\mathscr{F}^{-1} \circ \ell^k/k!$ is the projector to the eigenspace with the eigenvalue $N^{2g-k}$ by the action of $N^*$. This correspondence $\mathscr{F}$ is called Fourier transform.

Beauville proves that $\mathrm{CH}_s^p(X)$ is 0 for $s > p$ and for $s < p - g$, and conjectures that $\mathrm{CH}_s^p(X)$ is 0 also for $s < 0$. His conjecture holds for $p = 0, 1, g-2, g-1$ and $g$. So conjecturally, the possible eigenvalues in $\mathrm{CH}^p(X)$ are $N^p, N^{p+1}, \ldots, N^{2p}$.

The goal of this paper is to find analogous phenomena for the Chow rings of infinite symmetric products. When $X$ is a smooth connected variety with a fixed-point $P$ and $S^nX$ the $n$th symmetric product of $X$, adding the point $P$ determines the morphism $S^nX \to S^{n+1}X$. The infinite symmetric product of $X$ is the "direct limit" of this system of all the finite symmetric products, and we denote it by $S^\infty X$.

Unlike in the case of finite-dimensional algebraic varieties, there are (at least) two different notions of Chow groups, namely, Chow homology $\mathrm{CH}_* S^\infty X$ (classes of cycles of finite dimension) and Chow cohomology $\mathrm{CH}^* S^\infty X$ (classes of cycles of finite codimension). The Chow homology is the direct limit of Chow groups by pushforwards, graded by dimension, and the Chow cohomology is, roughly

Received 6 December 1993. Revision received 17 January 1996.