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KAZHDAN-LUSZTIG CONJECTURE FOR AFFINE LIE ALGEBRAS WITH NEGATIVE LEVEL II: NONINTEGRAL CASE

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To Professor Kiyosato Okamoto, who taught us the representation theory, in celebration of his sixtieth birthday

§0. Introduction. In [KT2], we proved G. Lusztig's conjecture [L2] concerning the Kazhdan-Lusztig type character formula for the irreducible highest weight modules over affine Lie algebras with negative level for the integral highest weight case. In this paper, we extend this result to the rational highest weight case. Our result, together with the results of Andersen-Jantzen-Soergel [AJS], Kazhdan-Lusztig [KL2], and Lusztig [L2], provides a proof of another conjecture of Lusztig [L1] concerning the characters of modular representations of finite Chevalley groups.

Let us state our results more precisely. Let $\mathfrak{g}$ be an affine Lie algebra, $\mathfrak{h}$ its Cartan subalgebra, $\{h_i\}_{i \in I}$ the set of simple coroots and W the Weyl group. Let $c \in \sum_{i \in I} \mathbb{Z}_{>0} h_i$ be a generator of the center of $\mathfrak{g}$. For $\lambda \in \mathfrak{h}^*$, let $M(\lambda)$ (resp., $L(\lambda)$) denote the Verma module (resp., irreducible module) with highest weight λ. Choose $\rho \in \mathfrak{h}^*$ such that $\langle \rho, h_i \rangle = 1$ for any $i \in I$. Our aim is to give a character formula for $L(w(\lambda + \rho) - \rho)$ for any $w \in W$ and any $\lambda \in \mathfrak{h}^*$ satisfying

$$\langle \lambda + \rho, h_i \rangle \in \mathbb{Q}_{\leqslant 0} \quad \text{for any } i \in I \quad \text{and} \quad \langle \lambda + \rho, c \rangle < 0. \tag{0.1}$$

Define the integral Weyl group $W(\lambda)$ for $\lambda \in \mathfrak{h}^*$ as the subgroup of W generated by the reflections with respect to the real coroots h satisfying $\langle \lambda, h \rangle \in \mathbb{Z}$. Then $W(\lambda)$ is naturally a Coxeter group (see §3.2 below). We denote its length function and its Bruhat order by l^λ and $\leqslant^\lambda$, respectively.

THEOREM 0.1. *Assume that $\lambda \in \mathfrak{h}^*$ satisfies (0.1). Let w be an element of W such that its length is smallest among the elements in the coset $wW(\lambda)$. For $y, x \in W(\lambda)$, let $P_{y,x}(q)$ be the Kazhdan-Lusztig polynomial for the Coxeter group $W(\lambda)$ (see [KL1]). Assume that $x \in W(\lambda)$ satisfies*

$$w'(\lambda + \rho) \neq wx(\lambda + \rho) \quad \textit{for any } w' \in W \textit{ with } w' < wx.$$

Then we have

$$\operatorname{ch} L(wx(\lambda + \rho) - \rho) = \sum_{y \leqslant^\lambda x} (-1)^{l^\lambda(x) - l^\lambda(y)} P_{y,x}(1) \operatorname{ch} M(wy(\lambda + \rho) - \rho).$$

Here ch *denotes the character.*

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