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ON THE ALGEBRAIC K-THEORY OF SIMPLY CONNECTED SPACES

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1. Introduction. This paper combines the cyclotomic trace invariant of [BHM] with the calculus of functors [G1] to evaluate Waldhausen's (reduced) functor $\tilde{A}(X)$ in terms of more familiar objects in algebraic topology, in the case of a simply connected X.

For each prime p, the cyclotomic trace gives a map of spectra

$$\mathrm{Trc}\colon A(X) \to \mathrm{TC}(X; p).$$

Here $A(X)$ denotes the version of Waldhausen's functor with $\pi_0 A(X) = Z$ rather than $\pi_0 A(X) = K_0(Z\pi_1 X)$.

There is a stable map from $\mathrm{TC}(X; p)$ to the suspension spectrum of the free loop space $\mathscr{L}X = \mathrm{Map}(S^1, X)$, with a disjoint base point added, and the composition with Trc is the topological Dennis trace of [B]. More generally, after p-adic completion the spectrum $\mathrm{TC}(X; p)$ was described completely in [BHM, Sect. 5]. The argument there is only correct in the case when $\pi_1 X$ is finite. However, the result is true in the general case by an argument due to Goodwillie. For a discussion of this, see [M]. We recall the result.

The self-maps of the circle, in particular, the rotation group S^1 and the degree p-map $\Delta_p(z) = z^p$, act on $\mathscr{L}X$ and therefore on the spectrum $\Sigma_+^\infty(\mathscr{L}X)$. After p-completion there is a fiber square (= homotopy cartesian diagram):

$$\begin{array}{ccc} \mathrm{TC}(X; p)_p^\wedge & \xrightarrow{\alpha} & \Sigma_+^\infty(S^1 \wedge (ES^1 \times_{S^1} \mathscr{L}X_+))_p^\wedge \\ \Big\downarrow{\scriptstyle\beta} & & \Big\downarrow{\scriptstyle\mathrm{Trf}} \\ \Sigma_+^\infty(\mathscr{L}X)_p^\wedge & \xrightarrow{1-\Delta_p} & \Sigma_+^\infty(\mathscr{L}X)_p^\wedge . \end{array} \tag{1.1}$$

The right-hand vertical map is the S^1-transfer and $\Sigma_+^\infty(Y)$ denotes the spectrum whose ith space is equal to

$$\lim_{\to} \Omega^n(S^{n+i} \wedge Y_+), \qquad Y_+ = Y \coprod \{+\}.$$

The composition of β and Trc is the p-completion of the topological Dennis trace.

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