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PERIODICITY AND THE MONODROMY THEOREM

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When Fox [F] asked in 1961 which knots can be fixed by periodic transformations of every order, an in-depth study of the infinitely equivariant properties of knots ensued. The first answers were given by Murasugi [M] for knots embedded in S^3, and by Sumners [S] in high dimensions. In this paper, we consider the following two questions: (1) which knots admit finite cyclic actions of every order, and (2) which knots admit finite cyclic actions of infinitely many orders. These problems are well understood for knots embedded in S^3 [M], [Fl], where the actions considered are either free or semifree. In high dimensions, Sumners [S] showed that if a spherical knot k is the fixed set of semifree $\mathbf{Z}_m$ actions for every m, then k must be the unknot. There is an equivalence [N2] between the existence problems for free versus semifree $\mathbf{Z}_m$ actions on odd-dimensional knots which are 1-simple, i.e., knots whose complements have fundamental group $\mathbf{Z}$. (There are, of course, no free $\mathbf{Z}_m$ actions, $m > 2$, on even-dimensional knots.) Thus, Sumners's unknotting theorem holds for free $\mathbf{Z}_m$ actions as well.

In this paper, we begin by giving a new proof of the unknotting theorem for the special case of odd-dimensional spherical simple knots, which are *high dimensional*, i.e., knots (S^{2n+1}, K^{2n-1}), $n > 2$, where $\pi_i(S - K) \cong \pi_i(S^1)$ for $i < n$, and K is a homotopy sphere. We then extend our result to include the larger class of simple fibered knots (S, K) where K is a rational homology sphere. All knots considered in this paper are smooth or PL.

UNKNOTTING THEOREM (2-1 and 2-3). *A simple knot k admits $\mathbf{Z}_m$-actions for all m if and only if it is the unknot.*

These actions are not assumed to be in any way compatible. The unknotting theorem answers a question closely related to problems studied by Weinberger [W]. There we find an analysis of whether the existence on a closed manifold of free $\mathbf{Z}_m$ actions for each m implies the existence of a $\mathbf{Q}/\mathbf{Z}$ action. In our case, the unknotting theorem shows that the existence of free $\mathbf{Z}_m$ actions for all m on the manifold pair (S^{2n+1}, K^{2n-1}) immediately gives the existence of compatible $\mathbf{Z}_m$ actions (i.e., of a $\mathbf{Q}/\mathbf{Z}$ action), in fact of an S^1 action; this, by virtue of (S, K) having to be the unknot. Our methods treat at once free and semifree actions. We then give a complete answer to the second question: which high-dimensional knots admit $\mathbf{Z}_m$ actions (free or semifree) for infinitely many m? Recall the definition for the rational monodromy h of a knot k: If $\tilde{X}$ is the infinite abelian cover and t the generator of the group of deck transformations, then a *rational mono-*

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