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GEOMETRY OF p-JETS

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Introduction. In a series of papers [B1–B4], we developed a differential algebraic method which was used, among others, to prove diophantine results over function fields. In this paper, we start developing an analogue of that method, which is designed to work over number fields. The main point in our differential algebraic method was the geometric study of "jet spaces" along a fixed derivation of the ground field. Of course, there is no nonzero derivation on a number field, so we cannot speak in the arithmetic context about usual jets. Instead of derivations, we will use certain natural nonadditive maps on rings of integers, which we call p-derivations. The resulting "jet spaces" will be called *p-jet spaces.*

Our paper has two parts. In the first part, we give a quick exposition of the theory in its simplest form: we shall only look at the geometry of "first order p-jets," and even this will be done in a special case. This case will be enough, however, to prove the following "quantitative version of the Manin-Mumford conjecture."

THEOREM A. *Let $X \to J$ be the Abel map defined over a number field K of a smooth curve of genus $g \geqslant 2$ into its Jacobian. Let $\wp$ be a prime of K with $p = \mathrm{char}\, \wp > 2g$. Assume that $K/\mathbf{Q}$ is unramified at $\wp$ and X/K had good reduction at $\wp$. Let K^a be the algebraic closure of K. Then*

$$\#(X(K^a) \cap J(K^a)_{tors}) \leqslant p^{4g} \cdot 3^g \cdot [p(2g-2) + 6g] \cdot g!.$$

A few remarks are in order. The finiteness of $\#(X(K^a) \cap J(K^a)_{tors})$ was conjectured by Manin and Mumford and first proved by Raynaud [Ray1]. In [Co1], Coleman considered the special case when J has complex multiplication, and, assuming in addition that X/K has ordinary reduction at $\wp$, he proved that $\#(X(K^a) \cap J(K^a)_{tors}) \leqslant p \cdot g$. (By the way, as shown by an example in [Co1], the bound $p \cdot g$ fails in general, in the noncomplex multiplication case.) Coleman's proof was based on his deep theory of p-adic abelian integrals [Co1] and heavily relies on both the complex multiplication assumption and the ordinarity assumption (cf. the discussion in [Co1, p. 157]).

Our strategy of proving Theorem A is the following. First, we prove a "nonramified version" of Theorem A: more precisely, we prove that Theorem A holds with K^a replaced by the maximal extension $K^{\wp}$ of K contained in K^a, which is unramified above $\wp$. Actually, such a result will be proved to hold without the

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