# CORE THEORY FOR MULTIPLE-SIDED ASSIGNMENT GAMES

WILLIAM F. LUCAS

**1. Introduction.** Game theory uses the methodologies of science and mathematics to analyze situations involving conflict and/or cooperation. Most game models are accordingly classified as either noncooperative or cooperative. In noncooperative games, each participant independently selects a specific course of action called a "pure strategy," with the goal to maximize gains. The main idea is that of a *mixed strategy*: a player picks a pure strategy according to a probability distribution and then evaluates payoffs statistically. The first great theorem in game theory was the minimax theorem of von Neumann [20]. This theorem proved the existence of an "optimal mixed strategy" for each player and a resulting (expected) value for any finite, two-person, zero-sum game, the so-called matrix games. Nash [10] generalized this theorem to prove the existence of optimal mixed strategies that realize an *equilibrium* outcome for the multiperson general-sum games. Most approaches to noncooperative models in game theory and mathematical economics today are still based upon Nash's contribution and its various extensions. Other papers in this volume illustrate this approach.

This paper is concerned primarily with cooperative games, in which the participants are allowed to communicate, form coalitions, make binding agreements, and redistribute their gains. Cooperative game theory with only two players is again a rather special case. The players must agree on how to select some Pareto optimal point from a region of all feasible outcomes. The first modern theory for selecting such a solution point is also due to Nash [9]. His work in this direction has led to a major branch in game theory known as the bargaining games involving two or more players. This approach is illustrated in the book by Roth [12].

Suggestions have been made over the years by von Neumann, Nash, Harsanyi, Weber [22], and others to the effect that cooperative games should be embedded into the noncooperative theory. Attempts to do so, however, have had rather limited success. On the other hand, it is often the case that many of the solution concepts for cooperative games do converge to the noncooperative solutions for several particular classes of games when the number of players becomes large. An example of this is the class of economic markets introduced by Shapley and Shubik [18].

When three or more players are engaged in a cooperative interaction, another phenomenon occurs. The players first consider what additional gains (or losses)

Received 23 January 1995.