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EIGENFUNCTION EXPANSION ASSOCIATED WITH THE CASIMIR OPERATOR ON THE QUANTUM GROUP $SU_q(1, 1)$

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1. Introduction and notations. The main purpose of this paper is to give the Plancherel formula for the quantum group $SU_q(1, 1)$ from the point of view of the spectral analysis of the Casimir operator.

Let us take the classical Lie group $SU(1, 1)$ of noncompact type as a typical example to explain the relationship between the spectrum of the Casimir operator and the regular unitary representations. As is well known, there are two kinds of irreducible regular unitary representations of $SU(1, 1)$: the principal continuous series and the discrete series. We can realize these two series as closed subspaces of $L^2(SU(1, 1))$ by using the spectral analysis for the Casimir operator in the following way.

Let C be the Casimir operator on the complex Lie group $SL(2, \mathbb{C})$ given by

$$C := FE + \frac{1}{4}H^2 + \frac{1}{2}H + \frac{1}{4}, \tag{1.1}$$

where E, F, and H are 2×2 matrices given by

$$E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad F = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

We see easily that Casimir operator C above is also well defined as the Casimir operator on the real form $SU(1, 1)$ of the complex Lie group $SL(2, \mathbb{C})$. Then, the following facts hold. (See Sugiura [Su].)

The Casimir operator C with the definition domain $C_0^\infty(SU(1, 1))$ has the unique selfadjoint extension. If we denote it by the same symbol C, then the spectrum $\sigma(C)$ of the selfadjoint operator C is given by

$$\sigma(C) = (-\infty, 0] \cup \left\{\frac{1}{4}k^2; k = 1, 2, 3, \ldots\right\}. \tag{1.2}$$

In (1.2), the eigenspace of the Casimir operator corresponding to the continuous spectrum $(-\infty, 0]$ stands for the principal continuous series of $SU(1, 1)$, and the

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