

SPECTRAL ESTIMATES AROUND A CRITICAL LEVEL

R. BRUMMELHUIS, T. PAUL, AND A. URIBE

1. Introduction and main statements	477
2. Oscillatory integral representations of spectral functions	485
3. A stationary phase formula	491
3.1. Preliminary expansions	
3.2. Case of Q indefinite	
3.3. Case of Q definite	
3.4. Proof of Theorem 1.1	
4. Near a critical energy level	502
4.1. The action-energy distribution $Y(s, E)$	
4.2. Y as a singular Lagrangian distribution	
4.3. The symbols of Y	
4.4. Proof of Theorem 1.2	
5. Small \hbar and small $ E - E_c $ estimates	511
6. A Tauberian lemma	515
6.1. Tauber with weights	
6.2. Proof of Theorem 1.3	
7. Eigenfunction estimates	519
Appendix. The density μ_t	522
A.1. The Duistermaat-Guillemin construction	
A.2. A special case	
A.3. Small t behavior	

1. Introduction and main statements. The semiclassical trace formula of [12], [3], [19], [20] is a rigorous version of the Gutzwiller trace formula which provides information about the spectral function of Schrödinger operators in the semiclassical regime. To be more specific, consider an operator of Schrödinger type on a compact manifold M , that is, an operator of the form

$$(1) \quad A_h = \sum_{j=0}^N \hbar^j A_j$$

where, for each j , A_j is a differential operator on M of order j . Define the Hamil-

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