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K_2 INVARIANTS OF 3-DIMENSIONAL PSEUDOISOTOPIES

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Let M^n be a compact topological manifold and let $C_{\mathrm{TOP}}(M)$ be its topological concordance space. This space arises in connection with a standard fibering of the homeomorphism group of M^n and has been studied extensively over the past 20 years (cf. [HW], [H], [I1, 2], [FJ], [KwS1]). In particular, the results of Hatcher, Wagoner, and Igusa led to a complete algebraic description of $\pi_0(C_{\mathrm{TOP}}(M^n))$, $n \geqslant 5$, in terms of higher algebraic K-theory. Namely, there is an exact sequence

$$(*) \quad K_3(Z[\pi]) \to Wh_1^+(\pi; \pi_2) \oplus Wh_1^+(\pi; Z_2) \to \pi_0(C_{\mathrm{TOP}}(M)) \xrightarrow{\sigma} Wh_2(Z[\pi]) \to 0,$$

where $\pi := \pi_1(M)$ and $\pi_2 := \pi_2(M)$.

In dimension three the algebraic invariants of Hatcher and Wagoner fail to give the complete description of $\pi_0(C_{\mathrm{TOP}}(M^3))$ [KwS1]. This failure is manifested in the existence of both stable and unstable pseudoisotopies of some 3-manifolds; i.e., the stabilization map

$$\Sigma^n: \pi_0(C_{\mathrm{TOP}}(M^3)) \to \pi_0(C_{\mathrm{TOP}}(M^3 \times I^n))$$

fails to be injective or surjective for these manifolds; in contrast, Σ^n is a bijection for $\dim M \geqslant 5$ and most likely for $\dim M = 4$ with some restriction on the fundamental group of M^4.

Although the algebraic invariants of Hatcher-Wagoner in general do not give the complete description of $\pi_0(C_{\mathrm{TOP}}(M^3))$, it turns out that some pseudoisotopies of M^3 are detected by these invariants (and hence they are stable; i.e., they survive under the suspension map Σ^n).

In particular the results of ([Kw], [KwS1]) provide a basic class of examples: *Let $x \to x^*$ be the canonical involution on $Wh_2(Z[\pi_1(M^n)])$ induced by conjugation in the group ring, and let M^3 be a closed 3-manifold whose fundamental group is small in the sense of Freedman and Quinn. Then the image* Im σ *of σ contains the subgroup $\mathscr{B}_-$ of all elements of the form $y - y^*$.*

This result, and the surjectivity of $\sigma: \pi_0(C_{\mathrm{TOP}}(M)) \to Wh_2(Z[\pi_1(M)])$ in higher dimensions led to the following problem.

PROBLEM 1. *Let M^3 be a closed, oriented, irreducible 3-manifold. Which of the elements in $Wh_2(Z[\pi_1(M^3)])$ are represented by pseudoisotopies of M^3?*

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