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SCATTERING THEORY FOR N-PARTICLE SYSTEMS IN CONSTANT MAGNETIC FIELDS

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1. Introduction. In this paper we study the behaviour of a system of N interacting particles in $\mathbf{R}^3$ in a constant magnetic field. Such a system is described by the Hamiltonian

$$H := \sum_1^N \frac{1}{2m_i}(D_i - q_i K x_i)^2 + \sum_{i<j} V_{ij}(x_i - x_j)$$

$$=: H_0 + V \text{ on } L^2(\mathbf{R}^{3N}),$$

where m_i and q_i are the mass and electric charge of the ith particle respectively, Kx is the vector potential associated with the magnetic field $\vec{B} = (0, 0, b)$, and V_{ij} is the interaction potential between the ith and jth particle. We do not require that q_i are nonzero, *i.e.*, some of the particles may be neutral.

Before we state our results rigorously, we would like to describe heuristically the new features of N-body scattering theory associated with the presence of an exterior magnetic field.

In the study of standard N-particle Hamiltonians (i.e., without an exterior magnetic field), it is customary to first separate the motion of the center of mass. The reason for this is that in this case H commutes with the translations

$$T_y u(x_1, \ldots, x_N) := u(x_1 - y, \ldots, x_N - y), \qquad y \in \mathbf{R}^3.$$

The generators of these translations are the components of the momentum of the center of mass of the system. When an exterior magnetic field is present, the translations T_y have to be replaced by *magnetic translations* whose infinitesimal generators are the components of the *pseudomomentum* (see Avron-Herbst-Simon [AHS2]). Unlike in the case when $\vec{B} = 0$, the components of the pseudomomentum do not commute except when the total charge of the system vanishes. When the total charge is nonzero, the pseudomomenta generate the product of the Heisenberg group in two dimensions with the translation group in one dimension. Thus the concept of separation of the center of mass is rather different in the presence of a magnetic field. Here we make the choice *not* to separate the motion

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