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NORMAL AND TANGENT RANKS OF CR MAPPINGS

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In recent years, several papers [B], [P1], [DF1], [BBR], [BR1], [BR2] have been published on the nonvanishing of the differential of CR mappings between hypersurfaces, in particular, on the nonvanishing of the derivative in a direction which is transverse to the complex tangent space. The first result on this subject seems to be in a paper of S. Pinchuk [P1] where this transversal derivative is estimated from below by means of the Hopf lemma.

We study here the higher-codimensional case. Nevertheless, even for hypersurfaces, our results are new or strictly generalize old ones.

Note with T_pM, the usual tangent space of a real manifold $M \subset \mathbb{C}^N$ at $p \in M$, by $T_p^cM \equiv T_pM \cap iT_pM$ its complex tangent space. M is said to be *generic* if $T_pM + iT_pM = \mathbb{C}^N$ holds for all $p \in M$. Each CR manifold is locally equivalent to a generic one. Note by $F_*: TM \to TM'$ the differential of a C^1 map $F: M \to M'$. Our main results are the following.

THEOREM 1. *Let $F: M \to M'$ be a CR mapping of class $C^{2,\alpha}$ between generic manifolds of class $C^{2,\alpha}$, $\alpha > 0$, and let p be a minimal point of M, such that $F_*(T_qM) + T^c_{F(q)}M' = T_{F(q)}M'$ holds for q arbitrarily close to p. Then $F_*(T_pM) \not\subset T^c_{F(p)}M'$ unless CR functions on M' extend holomorphically to a common full neighborhood of $F(p)$ in $\mathbb{C}^{N'}$. In any case, $F(-p)$ is minimal in M'.*

THEOREM 2. *Let $F: M \to M'$ be a homeomorphic CR mapping of class $C^{2,\alpha}$ between CR manifolds of class $C^{2,\alpha}$ with the same CR dimension. Then F is a diffeomorphism at all minimal points of M.*

Minimality is understood in the sense of Tumanov [T1].

THEOREM 3. *Let M, M' be CR manifolds of class $C^{2,\alpha}$, $\alpha > 0$, and $F: M \to M'$ a CR mapping of class $C^{2,\alpha}$. Suppose that, for some $p \in M$, $p' = F(p)$ is minimal and $F_*(T_p^cM) = T_{p'}^cM'$. Then F has surjective differential at p and p is minimal.*

We are grateful to an anonymous referee who pointed out to us an error in the proof of this theorem. We added Lemma 3 and the proof goes now on the same lines.

Theorem 3 permits us to give a definition of minimality of $p \in M$ which may be much easier to handle than the classic one [T1]. Suppose for simplicity that we

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