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RESIDUAL INTERSECTIONS AND SOME APPLICATIONS

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1. Introduction. In this paper, we give a new residual intersection decomposition for the refined intersection products of Fulton-MacPherson. Our formula refines the celebrated residual intersection formula of Fulton, Kleiman, Laksov, and MacPherson. The new decomposition is more likely to be compatible with the canonical decomposition of the intersection products and each term in the decomposition thus has simple geometric meaning. Our study is motivated by its applications to some geometric problems. In particular, we use the decomposition to find the distribution of limiting linear subspaces in degenerations of hypersurfaces. As another consequence, a family of identities for characteristic classes of vector bundles is also obtained.

Given a closed regular embedding of codimension d

$$i\colon X \to Y$$

and a morphism

$$f\colon V \to Y$$

with V a purely k-dimensional scheme, the fundamental construction of Fulton-MacPherson [F] defines the refined intersection product

$$X \cdot V \in A_{k-d}(W), \qquad W = f^{-1}(X),$$

where $A_{k-d}(W)$ is the $(k-d)$-th Chow group of W. Let N be the pull-back of the normal bundle $N(X, Y)$ of X in Y to W and $c(N)$ be its Chern class. Then $X \cdot V$ can be expressed in terms of $c(N)$ and the Segre class $s(W, V)$ of W in V by

$$X \cdot V = \{c(N) \cap s(W, V)\}_{k-d}.$$

Furthermore, there is a canonical decomposition

$$X \cdot V = \sum_j \alpha_j, \tag{1.1}$$

where α_j are classes supported on the so-called distinguished varieties Z_j of $X \cdot V$. It is well known that every irreducible component of W is a distinguished variety of

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