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GREEN FORMS AND THEIR PRODUCT

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Introduction. One of the basic ingredients of the Arakelov arithmetic intersection theory [A] is the Green function associated to a divisor in a Riemann surface from which the Arquimedian height pairing is defined. In order to extend the Arakelov theory to higher dimensions, Gillet and Soulé ([GS]) have generalized the concept of Green functions by introducing the notion of Green currents associated to an algebraic cycle on a complex manifold. If y and z are algebraic cycles and g_y and g_z are Green currents for y and z respectively, they have constructed a current $g_y * g_z$ which represents in some sense the intersection cycle $y \cdot z$. Moreover, when y and z intersect properly, they have proved that $g_y * g_z$ is a well-defined Green current for $y \cdot z$ and in this case the $*$-product is commutative, associative and gives the Arquimedian component of the higher height pairing. Note, however, that when the intersection is not proper, the current $g_y * g_z$ is not a Green current.

A key property of Green currents is that they can be represented by differential forms with suitable logarithmic singularities. This representability allows us to define inverse images of Green currents. This fact is essential in the definition of the $*$-product.

The aim of this paper, motivated by the work of Gillet and Soulé, is to attach to each cohomology class with support in a closed algebraic subset a space of Green forms extending the notion of Green currents associated with an algebraic cycle. Moreover we shall define a product between these spaces, which is commutative and associative even when the algebraic subsets do not intersect properly. We shall also call this product the $*$-product.

More precisely, let X be a complex projective manifold and let Y be a closed algebraic subset. We shall define a bigraded complex vector space $GE_{X,Y}^{*,*}$ provided with a real structure and with a real graded morphism

$$\mathrm{cl}\colon GE_{X,Y}^{*} \to H_Y^{*+2}(X, \mathbb{C}).$$

Using the mixed Hodge structure of $H_Y^{*+2}(X, \mathbb{C})$ we shall prove that this morphism is surjective.

For any two closed algebraic subsets Y and Z we shall define a $*$-product:

$$GE_{X,Y}^{p,q} \otimes GE_{X,Z}^{n,m} \xrightarrow{*} GE_{X,Y\cap Z}^{p+n+1,q+m+1},$$

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