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IDEALS OF MINORS IN FREE RESOLUTIONS

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Introduction. Let R be a commutative Noetherian ring, and let

$$\mathscr{F}: \cdots \to F_i \xrightarrow{\phi_i} F_{i-1} \to \cdots \to F_1 \xrightarrow{\phi_1} F_0 \to M \to 0$$

be a free resolution of a finitely generated R-module M with annihilator

$$\operatorname{ann} M = J.$$

It is interesting to ask how the invariants of the maps ϕ_i, such as the ideal $I_j(\phi_i)$ generated by the $j \times j$ minors of ϕ_i, reflect the properties of M. For example, it is not hard to show (see Buchsbaum-Eisenbud [4]) that if the grade of M is g (that is, g is the length of a maximal regular sequence contained in J) and r_i is the rank of the map ϕ_i (that is, the size of the largest nonvanishing minors of ϕ_i), then for $i < g$

$$\text{radical } J = \text{radical } I_{r_i}(\phi_i),$$

while at least in finite free resolutions, $I_{r_{g+1}}(\phi_{g+1})$ has strictly larger radical. For the case of ϕ_1, that is, for a free presentation of M, we have sharper estimates. For example, it is well known that

$$J^{r_1} \subseteq I_{r_1}(\phi_1) \subseteq J. \tag{1}$$

In this paper we will prove some results extending the left-hand inclusion in formula (1), and propose some conjectures which would extend the right-hand inclusion. We also study some related conjectures and results about annihilators of exterior powers of modules.

Huneke's Conjecture and its extensions. As is well known, the left-hand inclusion in formula (1) can be sharpened to a chain of inclusions,

$$JI_j(\phi_1) \subseteq I_{j+1}(\phi_1) \qquad \text{for } j < r_1. \tag{2}$$

(see Lemma 2.1).

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