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INEQUALITY OF BOGOMOLOV-GIESEKER TYPE ON ARITHMETIC SURFACES

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Introduction. Let M be an n-dimensional compact Kähler manifold with a Kähler form Φ. For a nonzero torsion free sheaf F on M, we define an average degree $\mu(F, \Phi)$ of F with respect to Φ by

$$\mu(F, \Phi) = \frac{\int_M c_1(F) \wedge \Phi^{n-1}}{\operatorname{rk} F}.$$

Let E be a nonzero torsion-free sheaf on M. E is said to be Φ-*stable* (resp. Φ-*semistable*) if, for every subsheaf F of E with $0 \subsetneq F \subsetneq E$, an inequality $\mu(F, \Phi) < \mu(E, \Phi)$ (resp. $\mu(F, \Phi) \leqslant \mu(E, \Phi)$) is satisfied. If a torsion-free sheaf E of rank r is Φ-semistable, then we have

$$\int_M \left(c_2(E) - \frac{r-1}{2r} c_1(E)^2 \right) \wedge \Phi^{n-2} \geqslant 0,$$

which is called the Bogomolov-Gieseker inequality (cf. [Bo] and [Gi]). The purpose of this paper is to establish an arithmetic analogue of the above inequality on an arithmetic surface.

Main Theorem. *Let K be an algebraic number field, O_K the ring of integers of K, and $f: X \to \operatorname{Spec}(O_K)$ an arithmetic surface. Let (E, h) be a Hermitian vector bundle on X. If $E_{\overline{\mathbb{Q}}}$ is semistable on the geometric generic fiber $X_{\overline{\mathbb{Q}}}$ of f, then we have an*

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