Vol. 74, No. 2 DUKE MATHEMATICAL JOURNAL © May 1994

ON THE TOPOLOGY OF NONNEGATIVELY CURVED SIMPLY CONNECTED 4-MANIFOLDS WITH CONTINUOUS SYMMETRY

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1. Introduction. Complete Riemannian manifolds with positive sectional curvature seem to be rare creatures in Riemannian geometry. Indeed, it is shown by D. Gromoll and W. Meyer [9] that a complete noncompact positively curved n-dimensional Riemannian manifold is diffeomorphic to the n-dimensional Euclidean space $\mathbf{R}^n$. On the other hand, every known example of compact positively curved manifold has a Lie group theoretic Riemannian metric with positive curvature and therefore a large group of isometry. A recent work of W. Y. Hsiang and B. Kleiner [13] shows that a 4-dimensional compact positively curved Riemannian manifold (henceforth abbreviated to **4DCPCM**) M with a nontrivial Killing vector field is homeomorphic to either S^4, $\mathbf{RP}^4$, or $\mathbf{CP}^2$, which are precisely the list of all known examples of **4DCPCM**. In the same paper, they also proposed the following two conjectures.

CONJECTURE 1. *A compact positively curved* 4*-dimensional Riemannian manifold with a nontrivial Killing vector field should be diffeomorphic to either* S^4, $\mathbf{RP}^4$, *or* $\mathbf{CP}^2$.

CONJECTURE 2. *A compact, simply connected, nonnegatively curved* 4*-dimensional Riemannian manifold with a nontrivial Killing vector field should be diffeomorphic to either* S^4, $\mathbf{CP}^2$, $S^2 \times S^2$, *or* $\mathbf{CP}^2 \# \pm\mathbf{CP}^2$.

Our first result in this paper is a topological classification of compact, simply connected, nonnegatively curved 4-dimensional Riemannian manifolds with a nontrivial Killing vector field. This partially solves the above Conjecture 2.

THEOREM 1. *Let M be a compact, simply connected, nonnegatively curved* 4*-dimensional Riemannian manifold with a nontrivial Killing vector field. Then M is homeomorphic to either* S^4, $\mathbf{CP}^2$, $S^2 \times S^2$, *or* $\mathbf{CP}^2 \# \pm\mathbf{CP}^2$. *Furthermore, if there is an effective isometric S^1-action on M such that its fixed point set $F(S^1, M)$ contains two* 2*-dimensional connected components, then M is diffeomorphic to* $S^2 \times S^2$ *or* $-\mathbf{CP}^2 \# \mathbf{CP}^2$.

Remark 1. The first two manifolds in the list are both symmetric spaces of rank one and have canonical metrics of positive curvature. $S^2 \times S^2$ with the product metric is of nonnegative curvature. The existence of nonnegatively curved

Received 22 June 1993. Revision received 1 November 1993.
Yang's research partially supported by NSF grant DMS 90-03524 and 9209330.