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CONNECTION FORMULA OF SYMMETRIC A-TYPE JACKSON INTEGRALS

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1. Introduction. Let $\overline{X} = \overbrace{C^* \times C^* \times \cdots \times C^*}^{n}$ be an algebraic torus of dimension n whose coordinates are represented by $t = (t_1, \ldots, t_n)$. We fix the elliptic modulus q such that $0 < q < 1$ and denote

$$X = \{q^\chi = (q^{\ell_1}, \ldots, q^{\ell_n}) | \chi = (\ell_1, \ldots, \ell_n) \in Z^n\}. \tag{1.1}$$

The set X is an integral lattice of $\overline{X}$ and acts on $\overline{X}$ by multiplication. For $\xi = (\xi_1, \ldots, \xi_n) \in \overline{X}$, we denote by $\langle \xi \rangle$ the X orbit of ξ

$$\langle \xi \rangle = \{q^\chi \xi = (\xi_1 q^{\ell_1}, \ldots, \xi_n q^{\ell_n}) | \chi = (\ell_1, \ldots, \ell_n) \in \mathbf{Z}^n\}. \tag{1.2}$$

Let $f(t)$ be a function on $\overline{X}$. We define the Jackson integral of $f(t)$ over $\langle \xi \rangle$ by

$$\begin{aligned} \int_{\langle \xi \rangle_+} f(t)\tilde{\omega} &= (1-q)^n \sum_{\chi \in \mathbf{Z}^n} f(q^\chi \xi) \\ &= (1-q)^n \sum_{\ell_1 = -\infty}^{+\infty} \cdots \sum_{\ell_n = -\infty}^{+\infty} f(\xi_1 q^{\ell_1}, \ldots, \xi_n q^{\ell_n}) \end{aligned} \tag{1.3}$$

where

$$\tilde{\omega} = \frac{d_q t_1}{t_1} \wedge \cdots \wedge \frac{d_q t_n}{t_n}.$$

We remark that in the previous papers [A1] and [AK] we use the notation $[0, \xi\infty]_q$ for $\langle \xi \rangle_+$. We endow such an orientation to $\overline{X}$ that the integrals are independent of the order of the variables $t_1, \ldots, t_n$.

In this paper, we choose the following function as an integrand:

$$\begin{aligned} \Phi_{n,m}(t) &= \Phi_{n,m}(t | \alpha, \beta, \gamma) \\ &= t_1^{\alpha_1} \cdots t_n^{\alpha_n} \prod_{\substack{1 \leqslant j \leqslant n \\ 1 \leqslant k \leqslant m}} \frac{(t_j/x_k)_\infty}{(t_j q^{\beta_k}/x_k)_\infty} \prod_{1 \leqslant i < j \leqslant n} \frac{(q^{\gamma'} t_j/t_i)_\infty}{(q^\gamma t_j/t_i)_\infty} \end{aligned} \tag{1.4}$$

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