Vol. 74, No. 1 DUKE MATHEMATICAL JOURNAL © April 1994

TOPOLOGICAL FINITENESS THEOREMS FOR MANIFOLDS IN GROMOV-HAUSDORFF SPACE

STEVEN C. FERRY

1. Introduction. In [GPW1], Grove, Petersen, and Wu used comparison theory and controlled topology to prove that for any given $n \neq 3$ there are at most finitely many homeomorphism types of Riemannian n-manifolds with any fixed lower bound on sectional curvature, lower bound on volume, and upper bound on diameter. One of the key steps in their argument is their proof that all of the manifolds in such a class have a common "contractibility function." Here is the definition.

Definition 1.1. A function $\rho\colon [0, R] \to [0, \infty)$, which is continuous at 0 with $\rho(0) = 0$ and $\rho(t) \geqslant t$ for all t, is a *contractibility function* for a metric space X if, for each $x \in X$ and $t \leqslant R$, the metric ball of radius t centered at x contracts to a point in the concentric ball of radius $\rho(t)$.

The purpose of this paper is to extend the work of Grove, Petersen, and Wu by investigating conditions under which collections of manifolds with a common contractibility function contain only finitely many homeomorphism types. The main theorem proves that this is true when $n \neq 3$ and the collection has compact closure in Gromov-Hausdorff space.

Definition 1.2. (i) If X and Y are compact subsets of a metric space Z, the *Hausdorff distance* between X and Y is

$$d_Z^H(X, Y) = \inf\{\varepsilon > 0 \mid X \subset N_\varepsilon(Y),\ Y \subset N_\varepsilon(X)\}.$$

(ii) If X and Y are compact metric spaces, the *Gromov-Hausdorff distance* from X to Y is

$$d_{GH}(X, Y) = \inf_Z\{d_Z^H(X, Y)\}$$

where X and Y are isometrically embedded in some Z.

(iii) Let $\mathscr{CM}$ be the set of isometry classes of compact metric spaces with the Gromov-Hausdorff metric.

Remark 1.3. (i) It is well known that $\mathscr{CM}$ is a complete metric space. See [G] or [P2] for an exposition.

Received 29 April 1993. Revision received 20 September 1993.
Author partially supported by NSF grant DMS 9003746. Author would like to thank the University of Chicago and the Institute for Advanced Study for support during the course of this work.