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HYPOELLIPTICITY IN THE TANGENTIAL CAUCHY-RIEMANN COMPLEX

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1. Introduction. In this paper we investigate the microlocal properties of the tangential Cauchy-Riemann complex of higher codimension. Here, we focus on the hypoellipticity of the operator $\bar{\partial}_b$ in the C^∞-category. Related microlocal questions of the tangential Cauchy-Riemann complex, specifically the hypoanalytic wavefront set and the holomorphic extendability of CR forms, are discussed in detail in [F2].

Let $\mathscr{D}$ be a domain in $\mathbf{C}^n$ containing the origin, while $\mathscr{B}$ stands for an open ball in $\mathbf{R}^d$ centered at 0. Throughout the whole paper the complex coordinates in $\mathbf{C}^{n+d}$, $n, d \geqslant 1$, are denoted by (z, w), where $z = (z_1, \ldots, z_n)$ and $w = (w_1, \ldots, w_d)$ with $x_j = \Re e z_j$, $y_j = \Im m z_j$, $j = 1, \ldots, n$, $s_k = \Re e w_k$, $k = 1, \ldots, d$.

A $(2n + d)$-dimensional smooth generic Cauchy-Riemann manifold $\mathscr{M}$ in $\mathbf{C}^{n+d}$ can be represented locally in the form

$$\mathscr{M} = \{(z, w) \in \mathbf{C}^{n+d}; \Im m w = \phi(z, \Re e w), z \in \mathscr{D}, \Re e w \in \mathscr{B}\} \tag{1}$$

with a real-valued C^∞ function $\phi(z, s) = (\phi_1(z, s), \ldots, \phi_d(z, s))$,

$$\phi|_0 = 0, d\phi|_0 = 0, \phi'_{ss}(0) = 0.$$

We shall say that the functions

$$z_j = x_j + iy_j, w_k = s_k + i\phi_k(z, s), \qquad j = 1, \ldots, n, k = 1, \ldots, d, \tag{2}$$

define a CR (Cauchy-Riemann) structure on $\mathscr{D} \times \mathscr{B}$.

Let $\mathscr{V}$ denote the vector subbundle of the complex tangent bundle $\mathbf{C}T(\mathscr{D} \times \mathscr{B})$ whose sections annihilate the functions z_j and w_k. The sections of $\mathscr{V}$ are spanned by the vector fields

$$L_j = \frac{\partial}{\partial \bar{z}_j} - i \sum_{k=1}^{d} \frac{\partial \phi_k}{\partial \bar{z}_j}(z, s) M_k, \tag{3}$$

where

$$M_k = \sum_{l=1}^{k} \mu_{lk}(z, s) \frac{\partial}{\partial s_l}, \qquad k = 1, \ldots, d, \tag{4}$$

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