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A NOTE ON NONVANISHING AND APPLICATIONS

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Introduction. Let $X \subset \mathbf{P}^N$ be a smooth projective variety of dimension n defined over the field of complex numbers. By L let us denote the restriction of $\mathcal{O}_{\mathbf{P}^N}(1)$ to X, and by K_X let us denote the canonical divisor of X (a linear equivalence class of the sheaf Ω_X^n of holomorphic n-forms). A (classical) version of *adjunction theory* concerns the studying of an *adjoint* linear system $|K_X + rL|$ for a suitably chosen positive integer r. In particular, a typical problem is to decide whether the *adjoint divisor* $K_X + rL$ is *nef* or for which value of r it becomes *ample* or even *very ample*. If $K_X + rL$ is *nef*, one may want to find out whether the linear system $|K_X + rL|$ has base points or for which positive integer m its multiple $|m(K_X + rL)|$ becomes base-point free.

Moreover, in the above situation, one may release the assumptions concerning L and X and ask all the above questons if L is merely ample and X is possibly singular. Problems concerning adjoint divisors have drawn a lot of attention to algebraic geometers, starting from the classical works of Castelnuovo and Enriques [CE], who considered adjoint linear systems on surfaces. Among the references on these problems, we would like to point out the ones which have inspired us the most: [BS1], [F3], [KMM], and the recent paper [K].

The most interesting case concerns the situation when the *adjoint divisor* $K_X + rL$ is *nef* but not ample. Then, although not much can be said about the system $|K_X + rL|$ itself, a Kawamata-Shokurov contraction theorem asserts that some of its multiple, $|m(K_X + rL)|$, is base-point free for $m \gg 0$ and defines an *adjoint contraction morphism* $\varphi: X \to Z$ onto a normal projective variety Z, with $\varphi_* \mathcal{O}_X = \mathcal{O}_Z$. Understanding this map seems to be very important for any classification theory of higher-dimensional manifolds.

In the present paper, we study the situation when L is merely ample and the *adjoint contraction morphism* has fibers of "small" dimension. This last hypothesis allows us to apply an inductive method which is typical of this theory, called "*Apollonius method*" by Fujita in [F3]. In the present paper we call it a *horizontal slicing* argument; it can be briefly summarized as follows. (Sometimes it is called simply *slicing*, but here we need to distinguish it from *vertical slicing*.)

Consider a general divisor X' from the linear system $|L|$ (a hyperplane section of X if L is very ample), and assume that it is a "good" variety of dimension $n - 1$ (i.e., it has the same singularities as X). By adjunction, $K_{X'} = (K_X + L)_{|X'}$, and by Kodaira-Kawamata-Viehweg vanishing theorem, if $r > 1$, the linear system $|m(K_{X'} + (r-1)L)|$ is just the restriction of $|m(K_X + rL)|$, so that the *adjoint con-*

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