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ON THE FUNDAMENTAL GROUP OF A COMPACT KÄHLER MANIFOLD

D. ARAPURA, P. BRESSLER, AND M. RAMACHANDRAN

0. Introduction. In this paper we prove that the fundamental group of a compact Kähler manifold cannot be an extension of a group with infinitely many ends by a finitely generated group. This implies that the fundamental group of such a manifold is indecomposable in a very strong sense. For example, it cannot be expressed as a nontrivial free product (or HNN extension) amalgamated over a finite group. The special case of free products had been established earlier by M. Gromov in [G4]. Our theorem is proved by a slight refinement of his methods. For the reader's convenience we have included careful proofs of some of Gromov's key results.

By a theorem of H. Hopf ([SW1]), the universal cover $\tilde{X}$ of a compact Kähler manifold X with infinite fundamental group has either one, two, or infinitely many ends. The main theorem of this paper rules out the possibility of $\tilde{X}$ having infinitely many ends. By standard considerations, if $\tilde{X}$ had two ends, then the first Betti number of X would equal one, which is impossible by Hodge theory. Thus $\tilde{X}$ must have one end. This gives an affirmative answer to a question posed by F.E.A. Johnson and E.G. Rees in [JR].

A synopsis of the paper is as follows. In the first section we recall the notion of L_2 cohomology in its various guises. In the next section we review various group theoretic notions and prove the nontriviality of the first L_2 cohomology space of a group with infinitely many ends. The third section is devoted to the key theorems on the existence of maps of complete Kähler manifolds with bounded geometry and nontrivial L_2 cohomology to Riemann surfaces. The main theorem is proved in the fourth section.

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1. L_2 cohomology. In this section we recall the definitions of reduced L_2 cohomology in the combinatorial and Riemannian contexts, following J. Dodziuk's exposition in [D].

Let G be a discrete group acting by simplicial orientation preserving automorphisms (respectively by orientation preserving isometries) and freely on a simplicial complex K (respectively oriented Riemannian manifold X) so that the quotient K/G is finite (respectively X/G is compact). We restrict our attention to this situation from now on.

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