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PSEUDODIFFERENTIAL OPERATORS ON GROUPS WITH DILATIONS

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1. Introduction. The theory of pseudodifferential operators had its origins in the work of Calderón-Zygmund [4] and Mikhlin [28]. They studied operators of the form

$$(\mathscr{K}f)(x) = \int_{\mathbb{R}^n} K(x, x - y)f(y)\, dy \tag{1}$$

so that

$$(\mathscr{K}f)(x) = (K_x * f)(x) \tag{2}$$

where $K_x(w) = K(x, w)$. Kohn and Nirenberg [25] rewrote $\mathscr{K}$ in the form

$$\begin{aligned}(\mathscr{K}f)(x) &= (2\pi)^{-n} \iint e^{-i(x-y)\cdot\xi} a(x, \xi) f(y)\, d\xi\, dy \\ &= \int e^{-ix\cdot\xi} a(x, \xi)\hat{f}(\xi)\, d\xi\end{aligned} \tag{3}$$

where formally,

$$\text{the inverse Fourier transform of } a \text{ in the second variable is } K. \tag{4}$$

There are obvious advantages to using (3), which have made it the generally preferred form. The Fourier transform converts convolution to a product, which is easier to handle. In particular, one can seek to use division to invert $\mathscr{K}$.

In problems where the Euclidean convolution structure is not relevant, these advantages are largely lost, and in many instances it is desirable to imitate the original definition of Calderón-Zygmund. For instance, if one is working on a Lie group, one can seek to define a class of pseudodifferential operators by (2), where

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