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SHARP PROPAGATION ESTIMATES FOR N-PARTICLE SYSTEMS

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1. Introduction. In this paper we establish sharp propagation estimates for N-particle Schrödinger Hamiltonians $H = \sum_1^N (-1/2m_i)\Delta_i + \sum_{i<j} V_{ij}(x_i - x_j)$ on $L^2(\mathbb{R}^{N_\nu})$. Propagation estimates are integral or pointwise estimates for large t on quantities of the form $\|B_t e^{-itH}\chi(H)\psi\|$ for some operator B_t. Integral estimates of the form

$$\int_{\mathbb{R}} \|B_t e^{-itH}\chi(H)\psi\|^2 \frac{dt}{\langle t\rangle} \leqslant C\|\psi\|^2 \tag{1.1}$$

are a basic ingredient of the proof of asymptotic completeness for short-range N-particle Hamiltonians by Sigal-Soffer [SS1] and Graf [Gr]. In these estimates, B_t can be a bounded pseudodifferential operator or a more general operator, and they carry the intuition that (in a rather weak sense) no propagation can take place on the "support" of B_t. We are interested here in proving pointwise estimates

$$\|B_t e^{-itH}\chi(H)\psi\| \leqslant C\langle t\rangle^{-k}\|\langle x\rangle^{-s}\psi\| \tag{1.2}$$

for some $k, s \in \mathbb{R}^+$. Sometimes this type of estimates can be obtained as a by-product of estimates on the boundary values of the resolvent $R(\lambda \pm i0)$, as for example in the works of Mourre [Mo] and Jensen [Je]. Recently, Signal and Soffer [SS2] have found an abstract way to prove directly pointwise estimates as (1.2). Their method relies on the method of positive commutators and on the construction of so-called *propagation observables*. They have obtained in this way minimal and maximal velocity estimates (see [SS2, Thms 3.1, 3.3]) and also a propagation estimate based on the generator of dilations $\frac{1}{2}(\langle x, D_x\rangle + \langle D_x, x\rangle)$, which is of a more microlocal nature.

In this paper we obtain new propagation estimates as (1.2). We will put a special emphasis in formulating the propagation estimates with pseudodifferential operators, which are closer to classical phase-space intuition. Let us comment a little on this point. In proving propagation estimates, it is technically convenient (and sometimes even necessary) to work with operators which are functions $F(t, A_t)$ of the time variable t and of a given selfadjoint differential or pseudodifferential operator A_t. Typically, one obtains propagation estimates of the form

$$\|F(t, A_t)e^{-itH}\chi(H)\psi\| \leqslant C\langle t\rangle^{-k}\|\langle x\rangle^{-s}\psi\|.$$

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